

Fourier optics



15-463, 15-663, 15-862
Computational Photography
Fall 2018, Lecture 24

Course announcements

- Any questions about homework 6?
- If you haven't already, please sign up for project checkpoint meetings.

Overview of today's lecture

- The scalar wave equation.
- Basic waves and coherence.
- The plane wave spectrum.
- Fraunhofer diffraction and transmission.
- Fresnel lenses.
- Fraunhofer diffraction and reflection.

Slide credits

Some of these slides were directly adapted from:

- Anat Levin (Technion).

Scalar wave equation

Simplifying the EM equations

Scalar wave equation:

- Homogeneous and source-free medium
- No polarization

$$\left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) u(r, t) = 0$$



speed of light in medium

Simplifying the EM equations

Helmholtz equation:

- Either assume perfectly monochromatic light at wavelength λ
- Or assume different wavelengths independent of each other

$$(\nabla^2 + k^2)\psi(r) = 0$$

$$u(r, t) = \text{Re} \left\{ \psi(r) e^{-j \frac{2\pi c}{\lambda} t} \right\}$$

 what is this?

$$\psi(r) = A(r) e^{j\varphi(r)}$$

Simplifying the EM equations

Helmholtz equation:

- Either assume perfectly monochromatic light at wavelength λ
- Or assume different wavelengths independent of each other

$$(\nabla^2 + k^2)\psi(r) = 0$$

Wave is a sinusoid at frequency $2\pi/\lambda$:

$$u(r, t) = \text{Re} \left\{ \psi(r) e^{-j \frac{2\pi c}{\lambda} t} \right\}$$

$$\rightarrow \psi(r) = A(r) e^{j\varphi(r)}$$

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Simplifying the EM equations

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At every point, wave has amplitude $A(r)$ and phase $\varphi(r)$:

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Simplifying the EM equations

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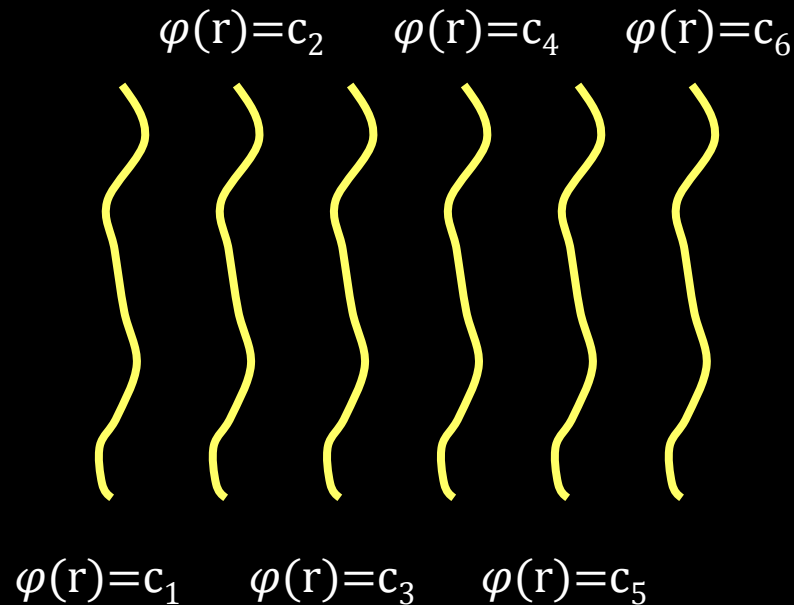
This is how we will describe waves for the rest of lecture

Basic waves and coherence

Visualizing a wave

Wavefront: A set of points that have the same phase

- Points on the wavefront have “travelled” the same distance from wave source
- Gives us “shape” of the wave



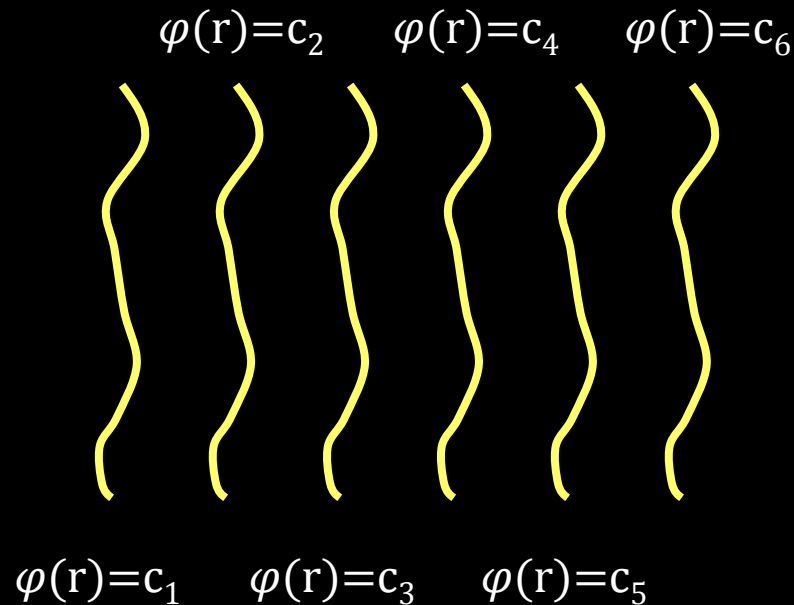
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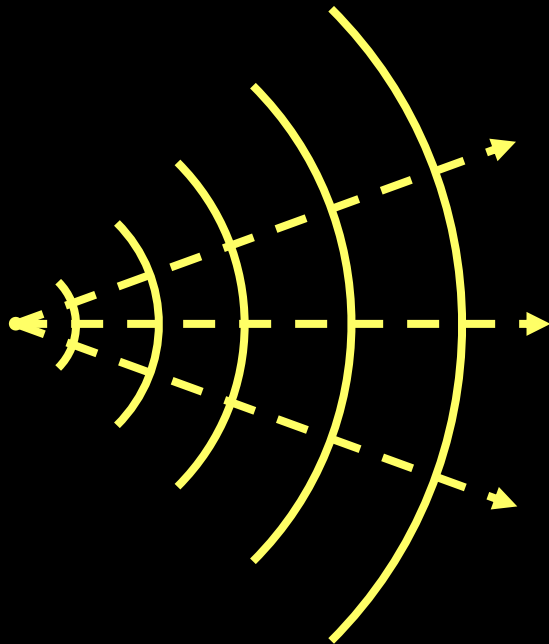
Roughly speaking,
in ray optics we
replace waves
with “rays” that
are always normal
to wavefront

At every point, wave has amplitude $A(r)$ and phase $\varphi(r)$:

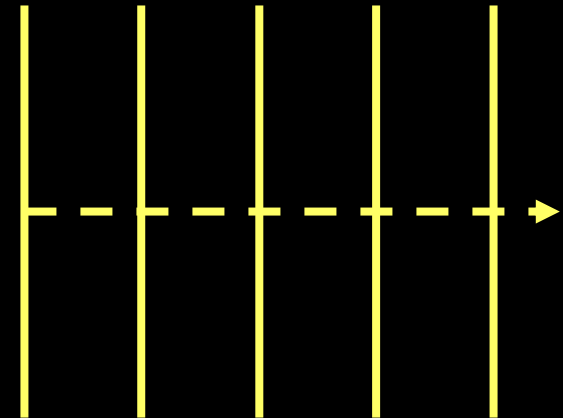
$$\psi(r) = A(r)e^{j\varphi(r)}$$

Two important waves

Spherical wave



Plane wave

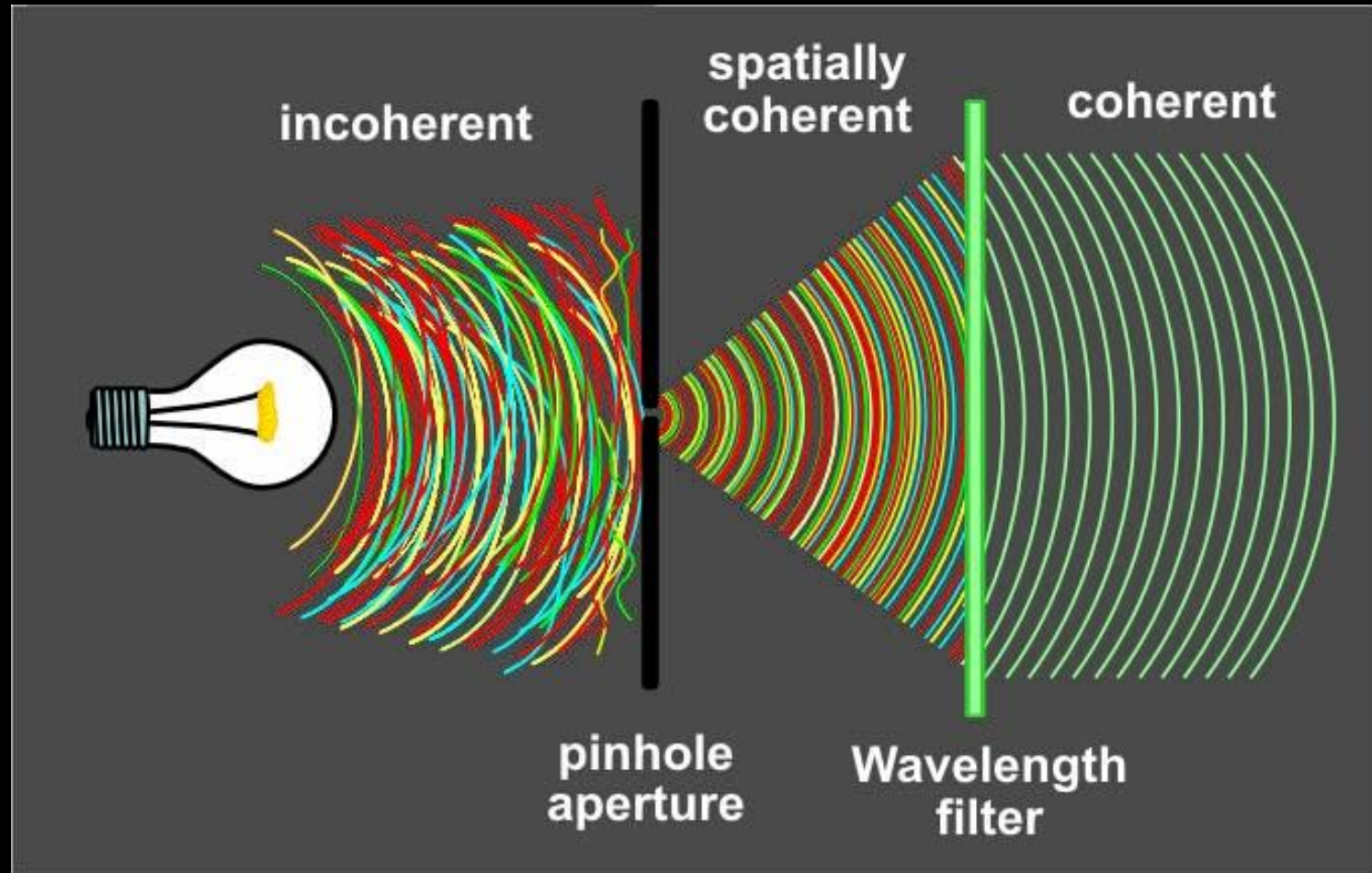


How can you create
a spherical wave?

At every point, wave has amplitude $A(r)$ and phase $\varphi(r)$:

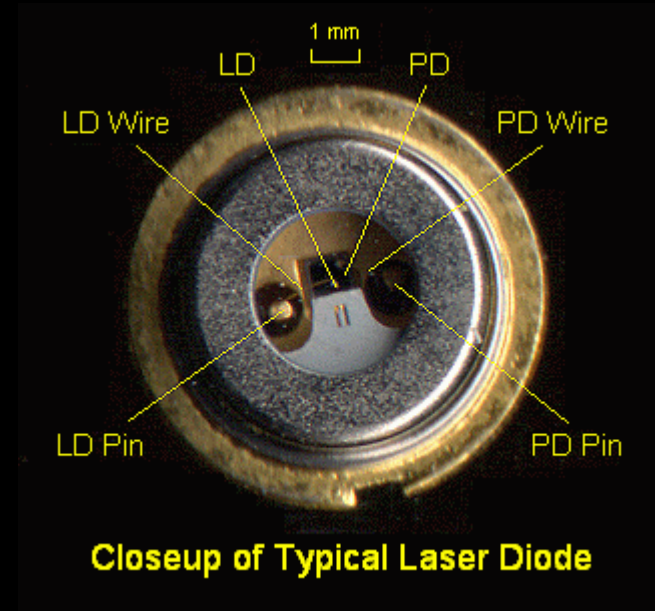
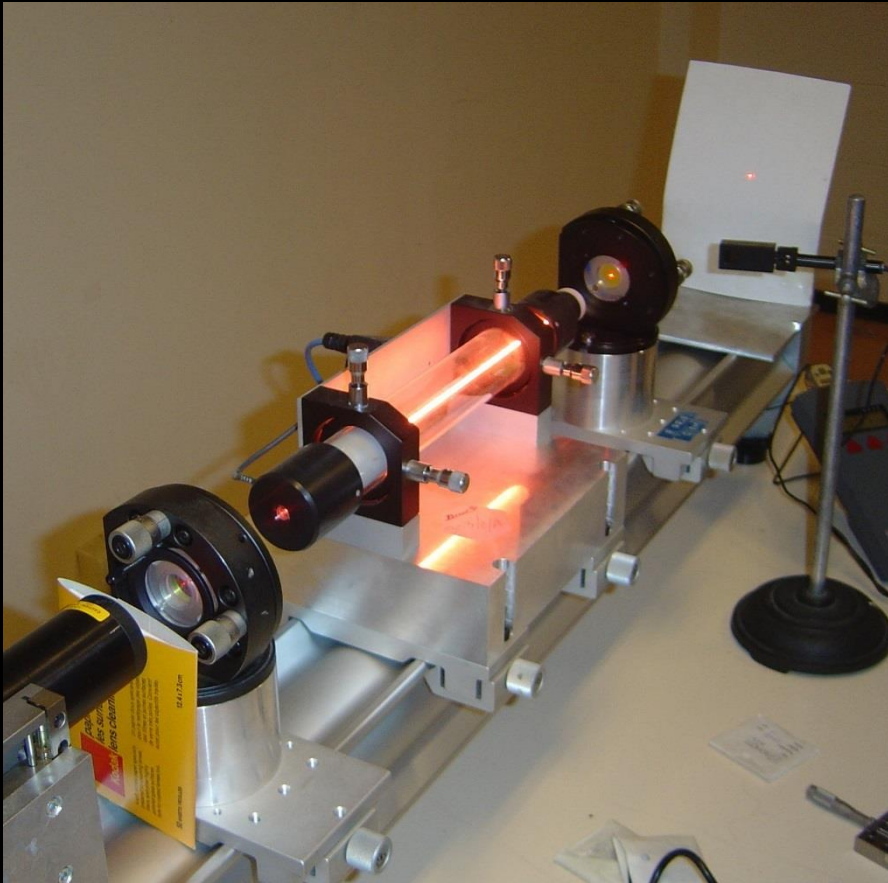
$$\psi(r) = A(r)e^{j\varphi(r)}$$

Creating a spherical wave using pinholes



- Any problems with this procedure?
- Do you know of any alternatives?

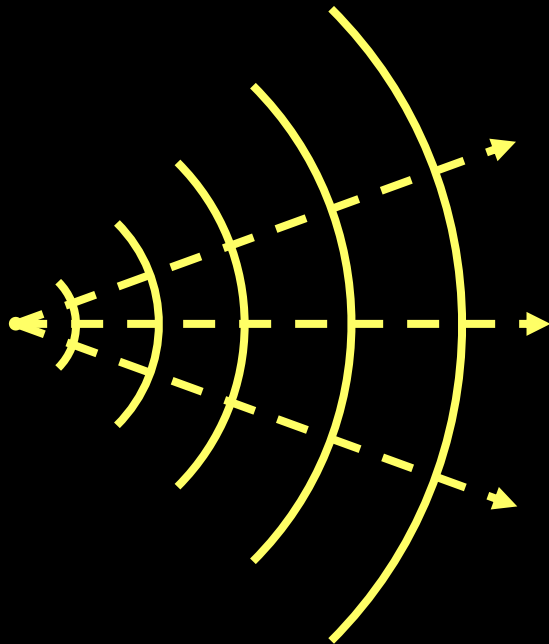
Creating a spherical wave using lasers



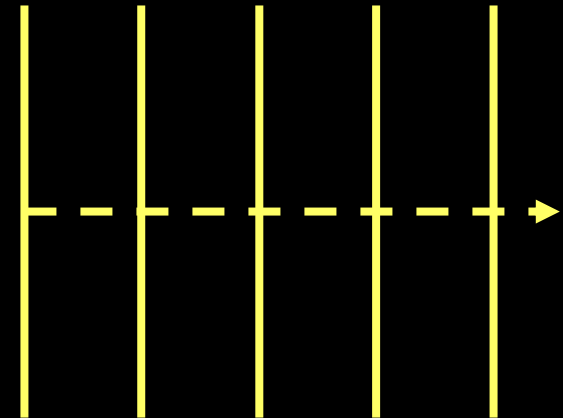
- Lasers are high-power “point” sources
- Standard lasers are also monochromatic (temporally coherent)

Two important waves

Spherical wave



Plane wave



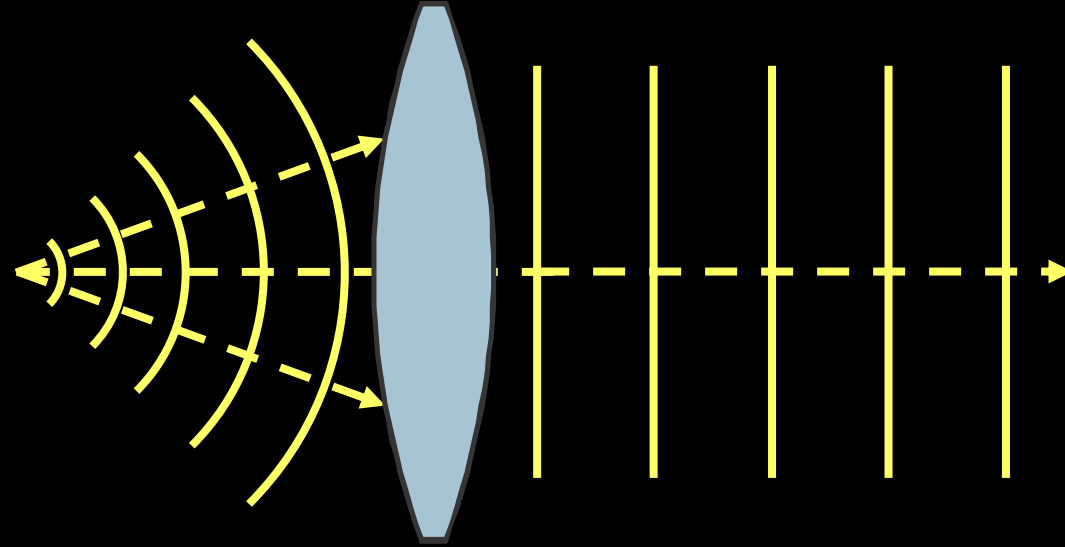
How can you create
a plane wave?

At every point, wave has amplitude $A(r)$ and phase $\varphi(r)$:

$$\psi(r) = A(r)e^{j\varphi(r)}$$

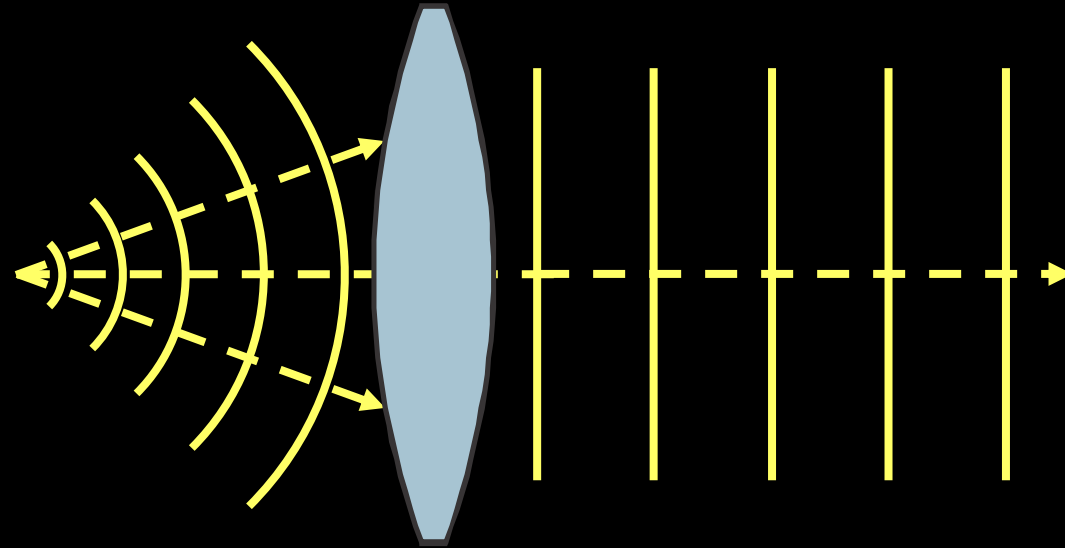
Creating plane waves

1. Use a thin lens:



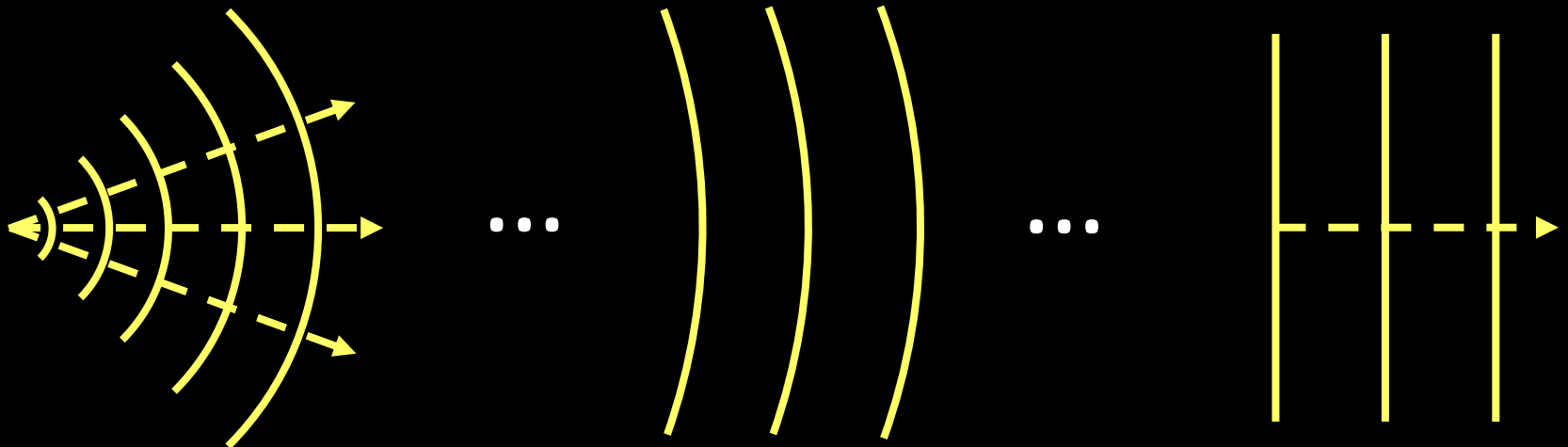
Creating plane waves

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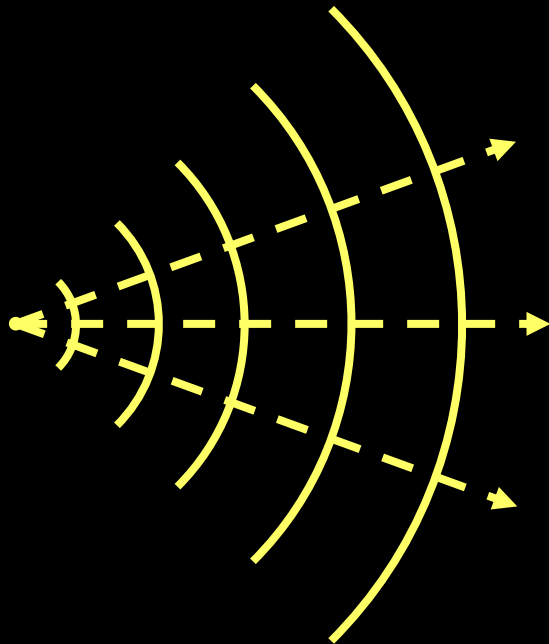
2. Let a spherical wave propagate a very long distance:

- This is often called the “far-field” assumption.

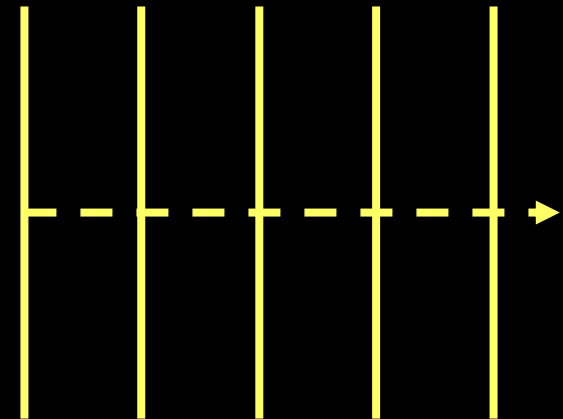


Two important waves

Spherical wave



Plane wave



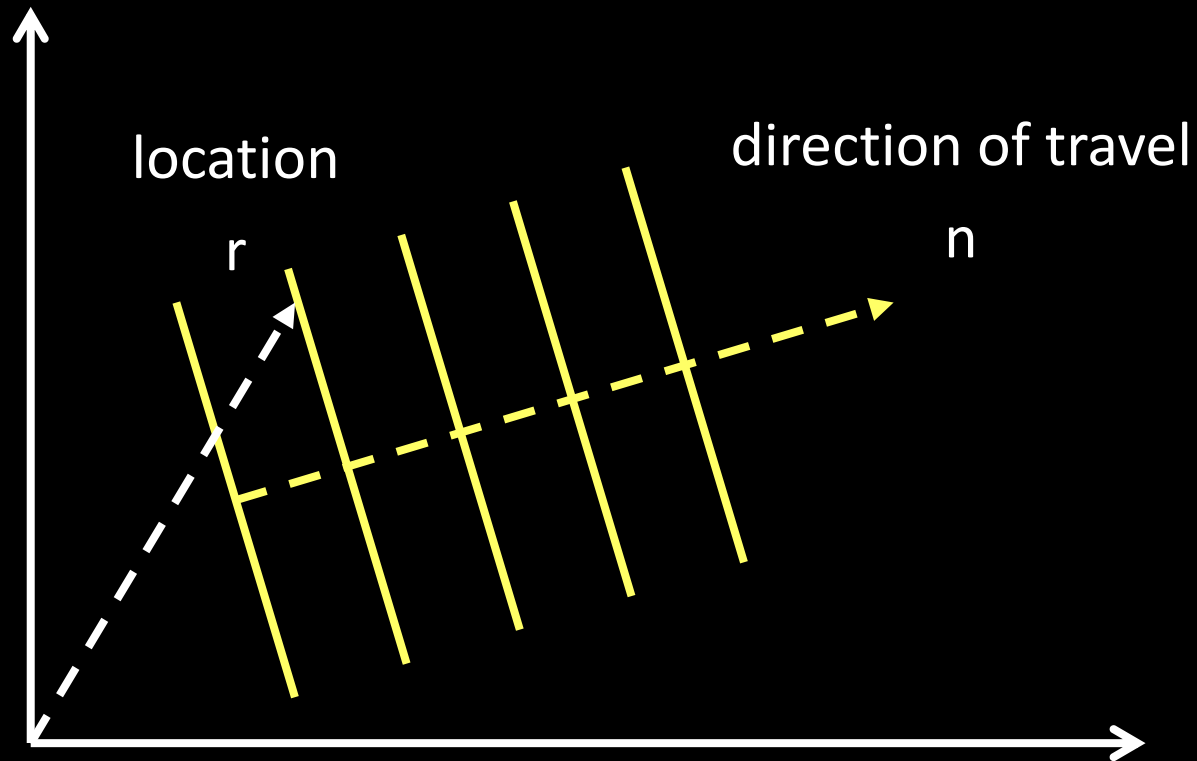
What is the equation of a plane wave?

At every point, wave has amplitude $A(r)$ and phase $\varphi(r)$:

$$\psi(r) = A(r)e^{j\varphi(r)}$$

The plane wave spectrum

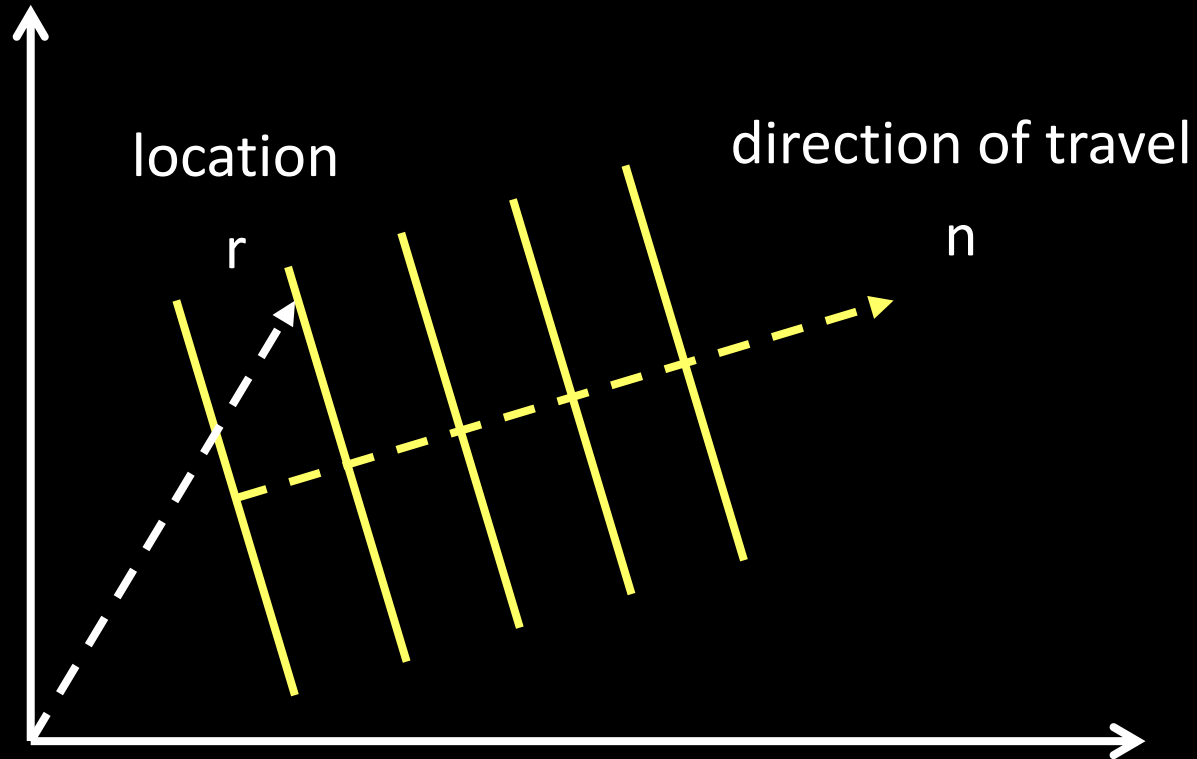
Plane wave equation



At every point, wave has amplitude $A(r)$ and phase $\varphi(r)$:

$$\psi(r) = A(r)e^{j\varphi(r)}$$

Plane wave equation



Plane wave equation:

$$\psi_{p,k}(r) = e^{jk \cdot r}$$

Wave vector:

$$k = \frac{2\pi c}{\lambda} n$$

does this remind you of something?

Plane wave spectrum

Every wave can be written as the weighted superposition of planar waves at different directions

$$\psi(r) = A(r)e^{j\varphi(r)}$$

$$\psi(r) = \int_{\vec{k}} \Psi(k) \psi_{p,k}(r) dk$$

$$\psi(r) = \int_{\vec{k}} \Psi(k) e^{jk \cdot r} dk$$

How are these weights determined?

Plane wave spectrum

Every wave can be written as the weighted superposition of planar waves at different directions

$$\psi(r) = A(r)e^{j\varphi(r)}$$

$$\psi(r) = \int_{\vec{k}} \Psi(k) \psi_{p,k}(r) dk$$

$$\psi(r) = \int_{\vec{k}} \Psi(k) e^{jk \cdot r} dk$$

$$\Psi(k) = \text{Fourier}\{\psi(r)\}$$

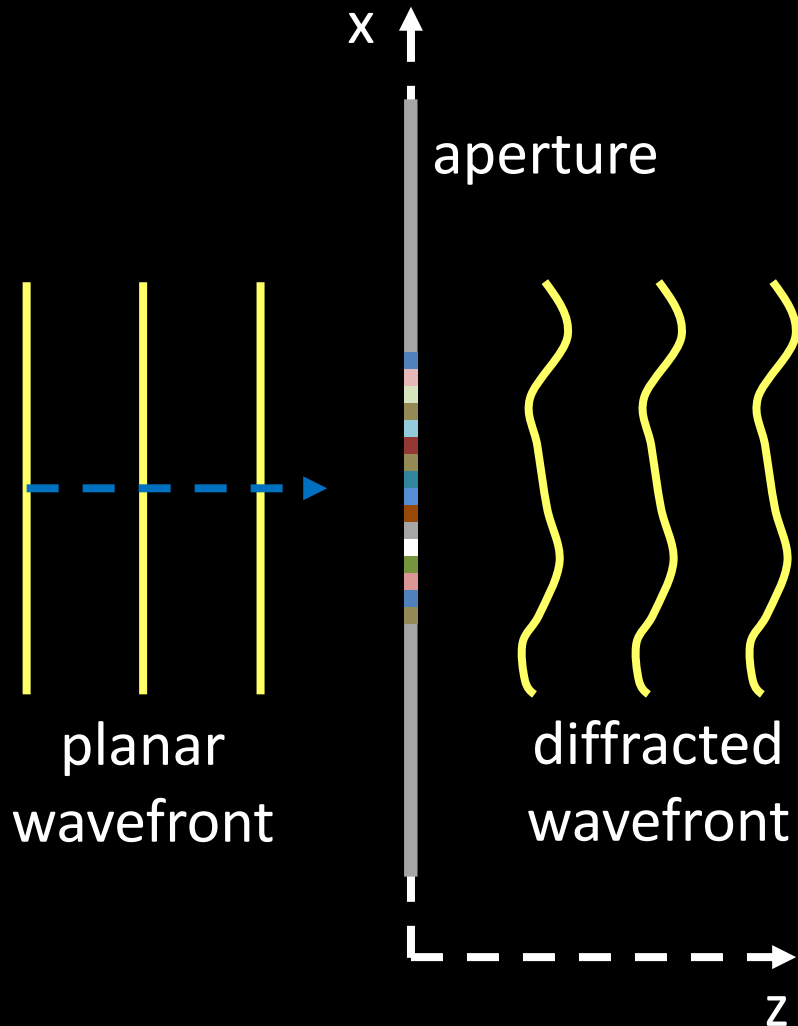
This is the wave's plane wave spectrum

Fraunhofer diffraction and transmission

Fraunhofer diffraction

Wave-optics model for transmission through apertures

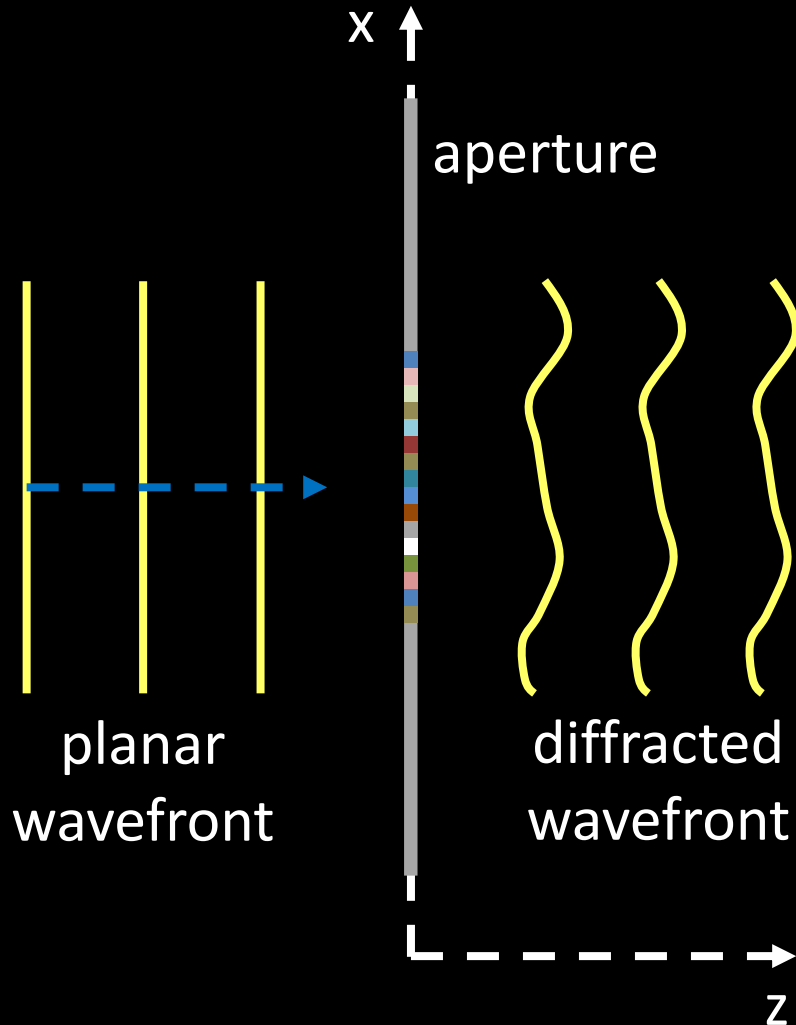
- Far-field assumption: Light is coming from and measured



Fraunhofer diffraction

Wave-optics model for transmission through apertures

- Far-field assumption: Light is coming from and measured



- transmission function:

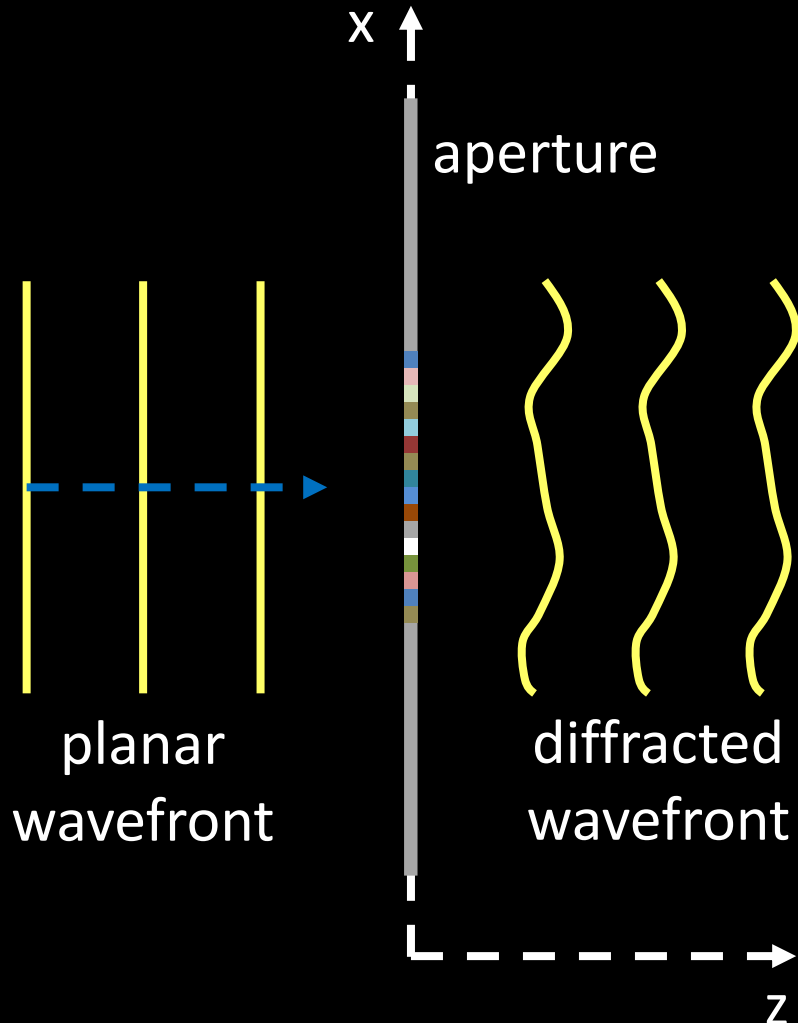
$$p(r) = A(r) \cdot \exp(j \cdot \Phi(r))$$

amplitude modulation phase modulation

Fraunhofer diffraction

Wave-optics model for transmission through apertures

- Far-field assumption: Light is coming from and measured



- transmission function:

$$p(r) = A(r) \cdot \exp(j \cdot \Phi(r))$$

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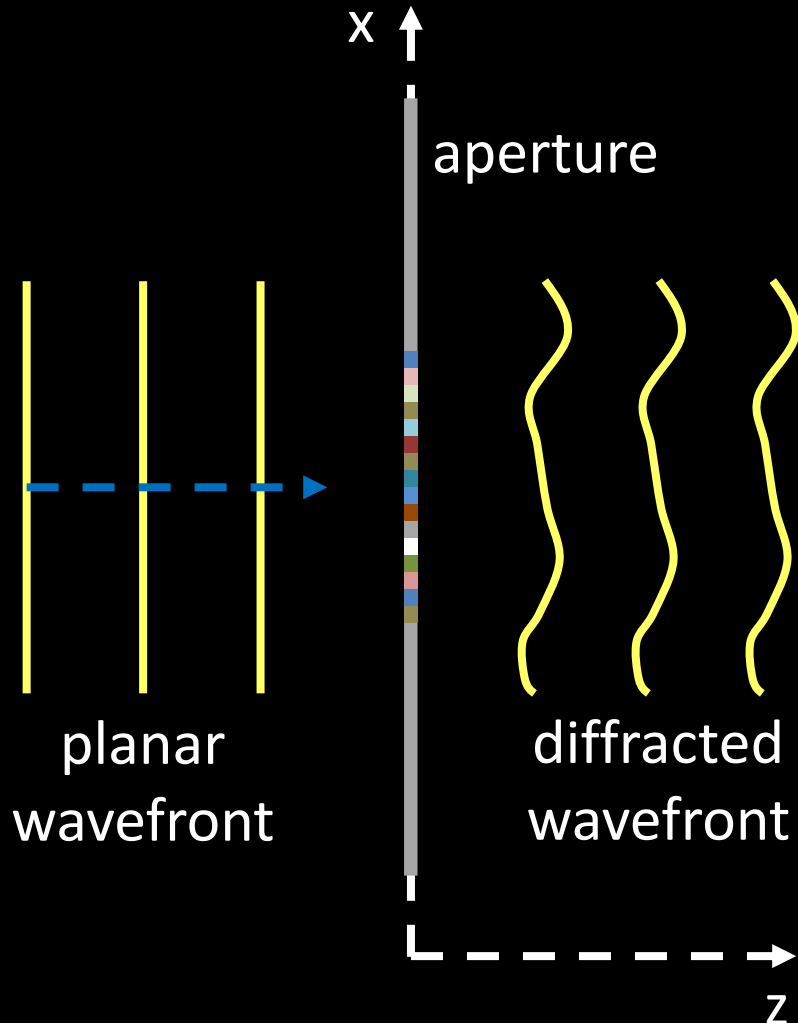
- transfer function:

$$P(k) = \text{Fourier}\{p(r)\}$$

Fraunhofer diffraction

Wave-optics model for transmission through apertures

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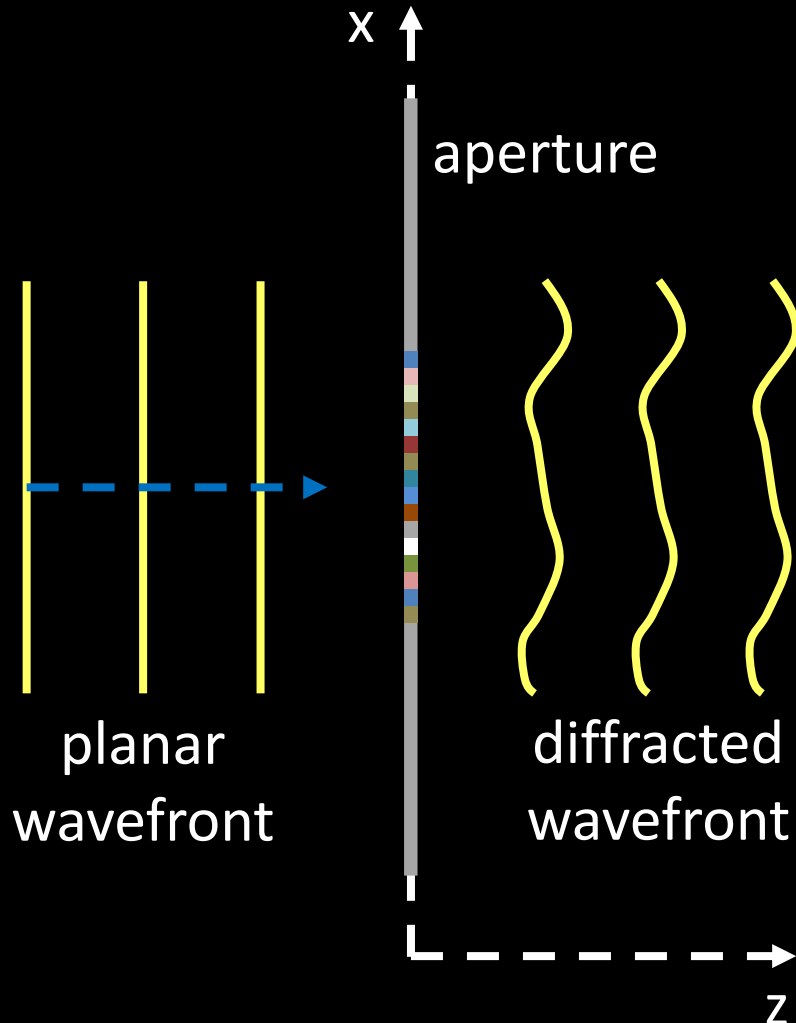
- plane spectrum of outgoing wave:

$$\Psi_{\text{out}}(k) = P(k) \cdot \Psi_{\text{in}}(k)$$

Fraunhofer diffraction

Wave-optics model for transmission through apertures

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- transmission function:

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amplitude modulation phase modulation

- transfer function:

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- plane spectrum of outgoing wave:

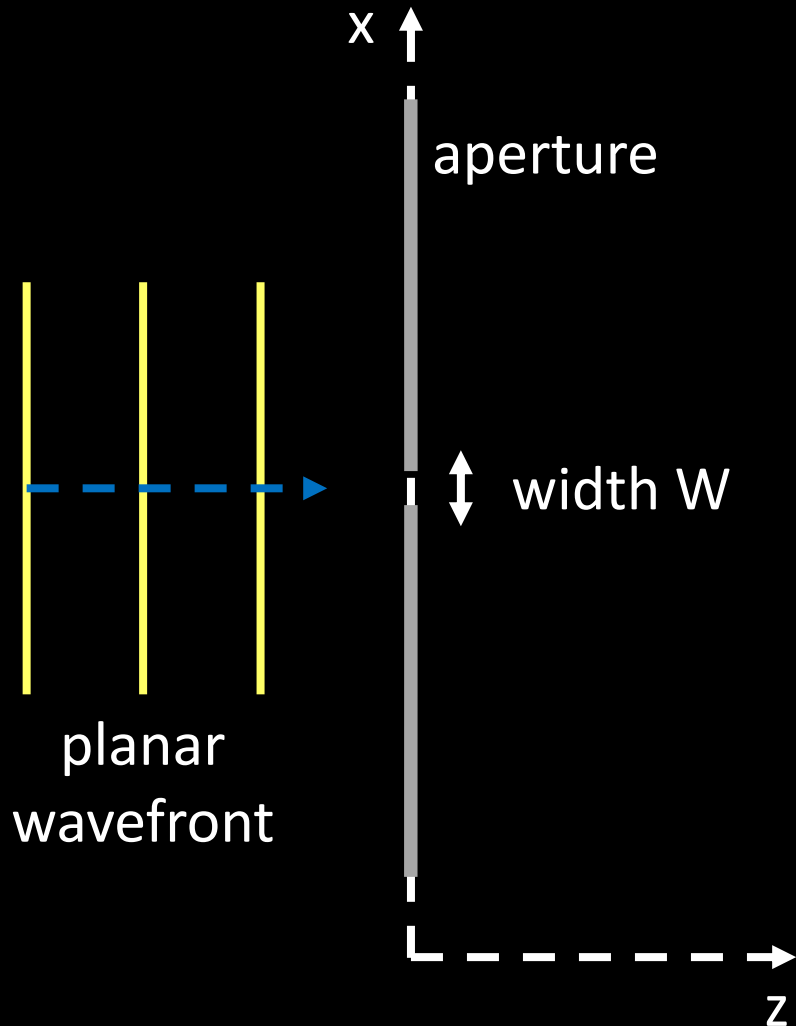
$$\Psi_{\text{out}}(k) = P(k) \cdot \Psi_{\text{in}}(k)$$

- outgoing wave:

$$\psi_{\text{out}}(r) = \text{Fourier}^{-1}\{ \Psi_{\text{out}}(k) \}$$

Example: pinhole

What is the transmission function?

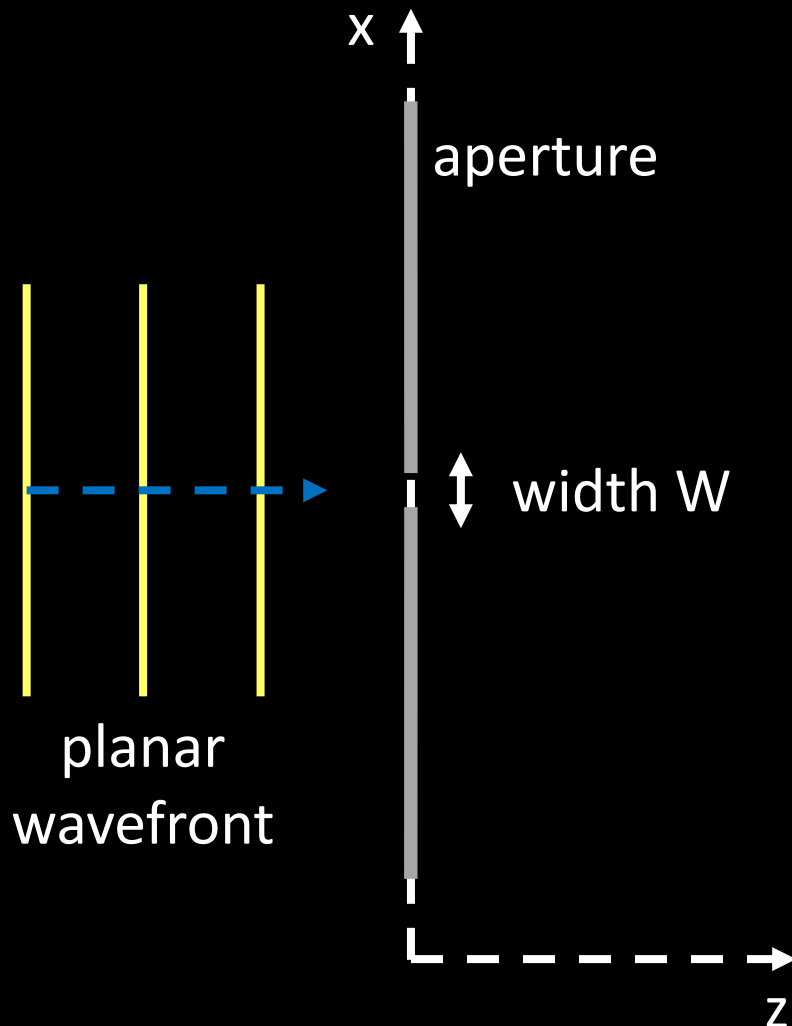


Example: pinhole

What is the transmission function?

$$p(r) = \text{rect}(W \cdot r)$$

What is the transfer function?



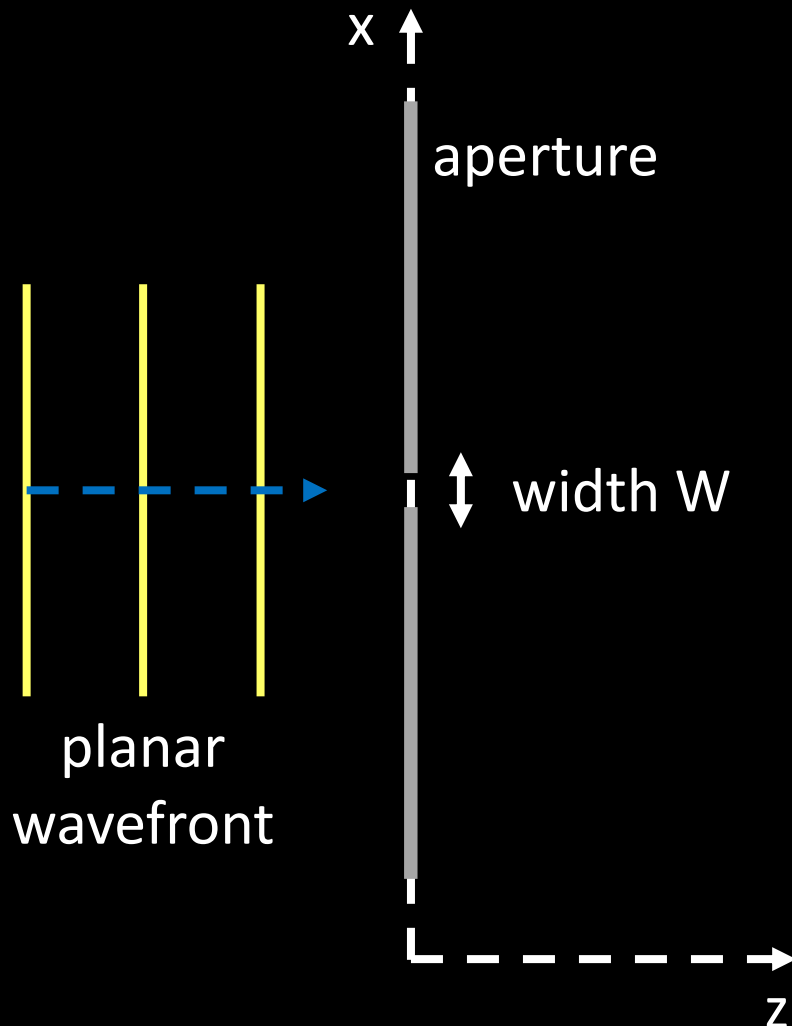
Example: pinhole

What is the transmission function?

$$p(r) = \text{rect}(W \cdot r)$$

What is the transfer function?

$$P(k) = \text{sinc}(k / W)$$



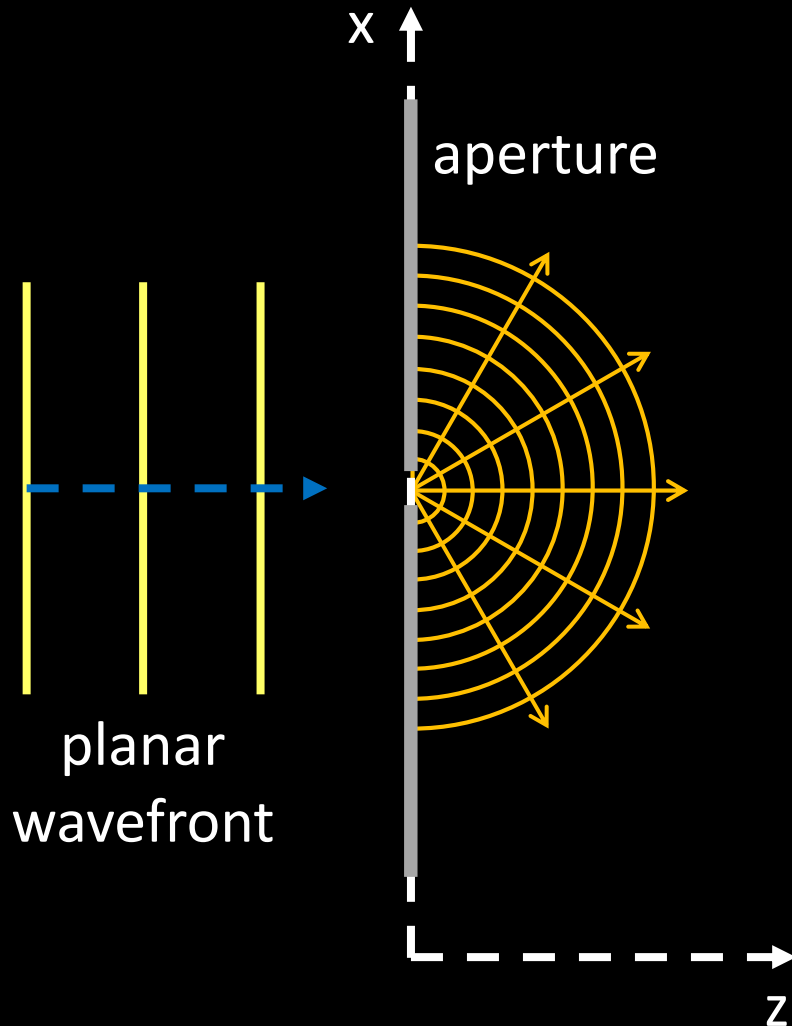
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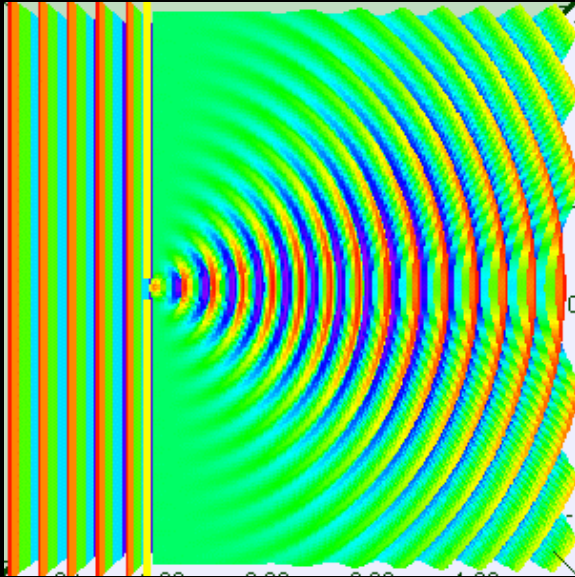
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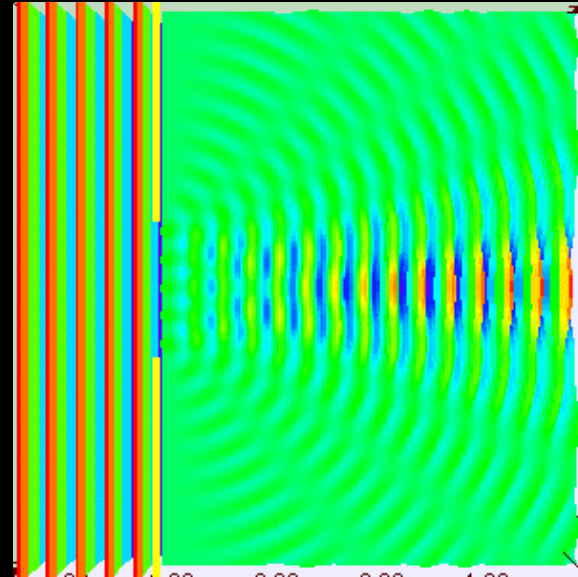
Example: pinhole

Why does the diffraction pattern become wider as we increase width?



small pinhole

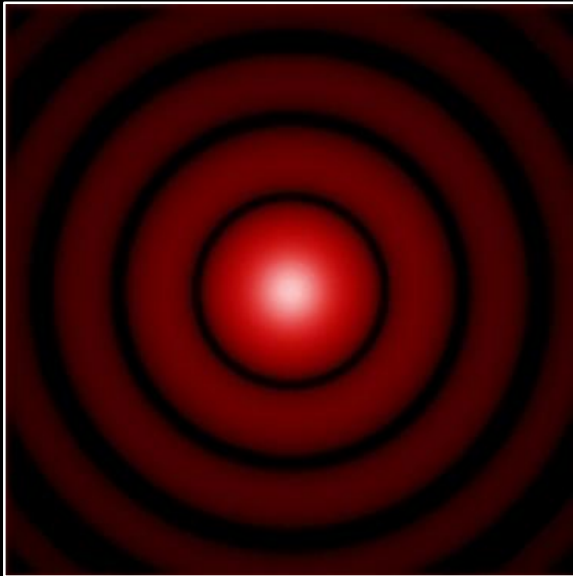
wide
diffraction
pattern



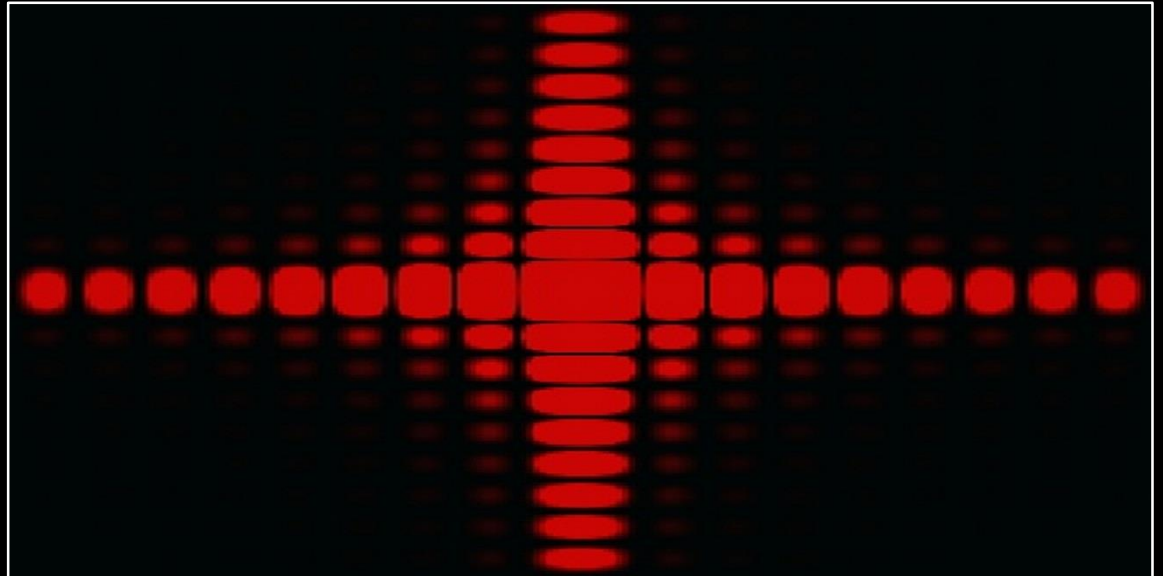
large pinhole

narrow
diffraction
pattern

Remember: 2D Fourier transform



circular aperture
(Airy disk)

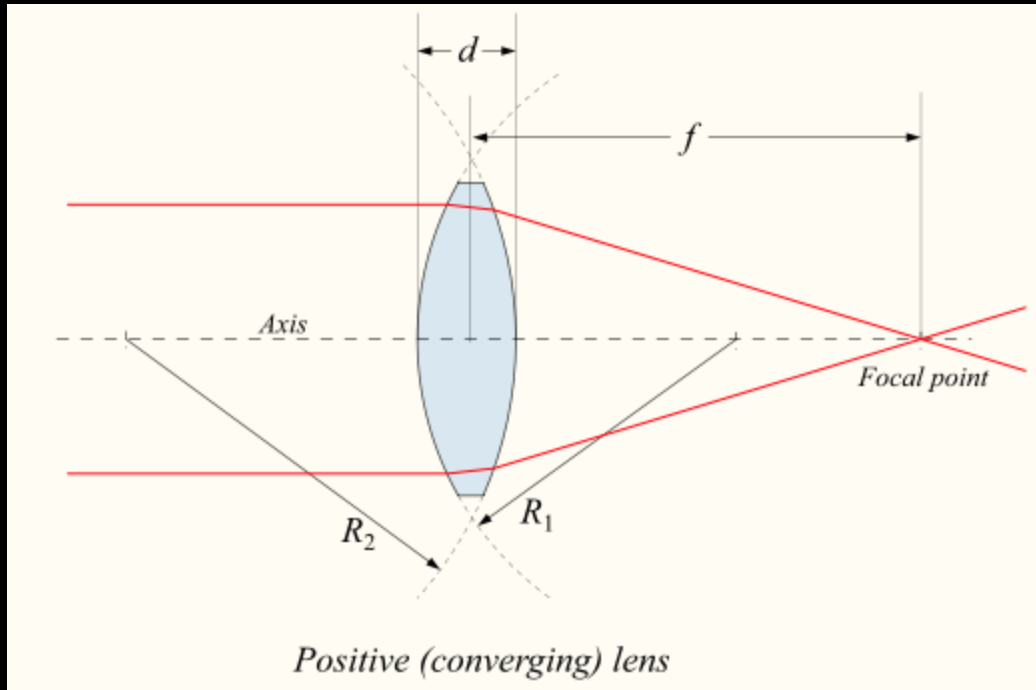


rectangular aperture

Fresnel lenses

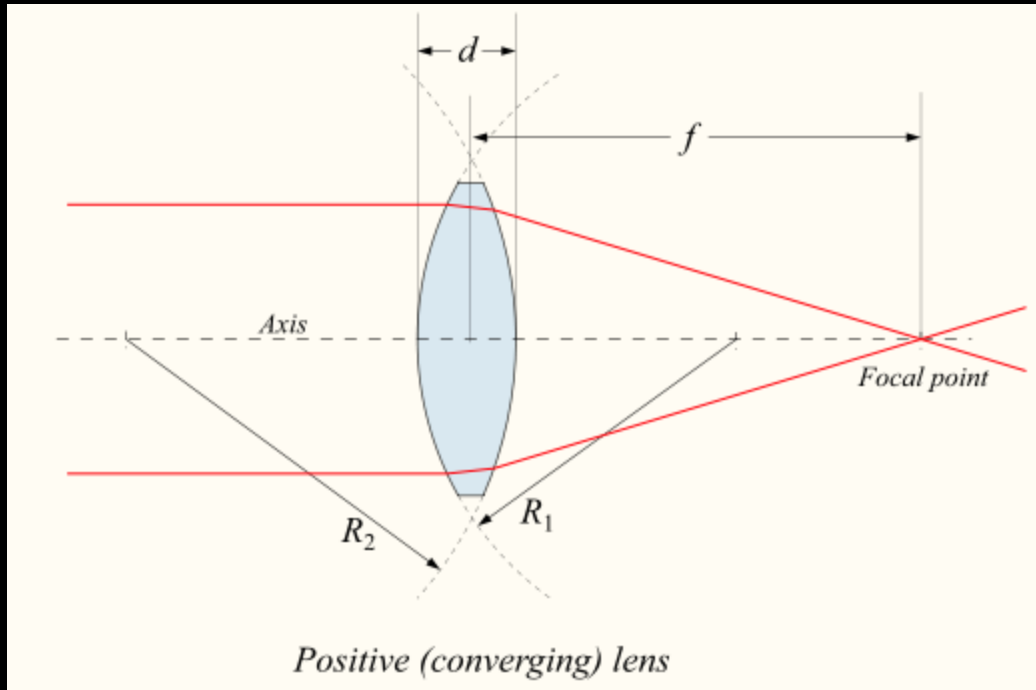
Thin lenses

What is the transmission function of a thin lens?



Thin lenses

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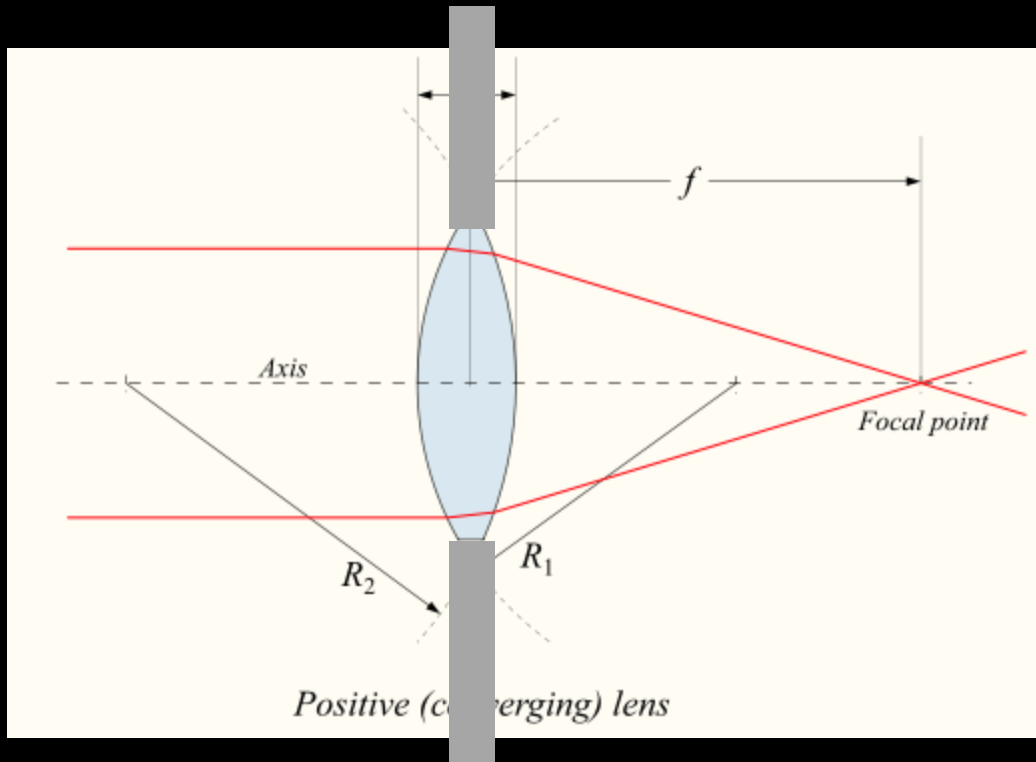


Complicated expression, but phase-only: $p(r) = \exp(j \cdot \Phi(r))$

- Delay all plane waves so that they have the same phase at focal point

Thin lenses

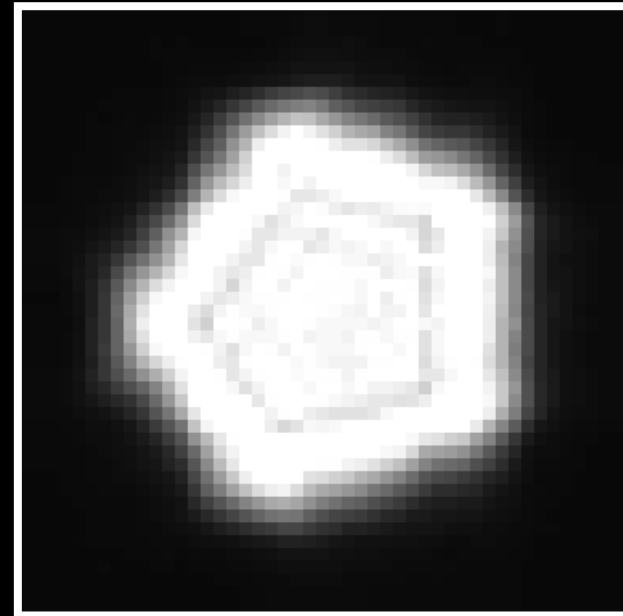
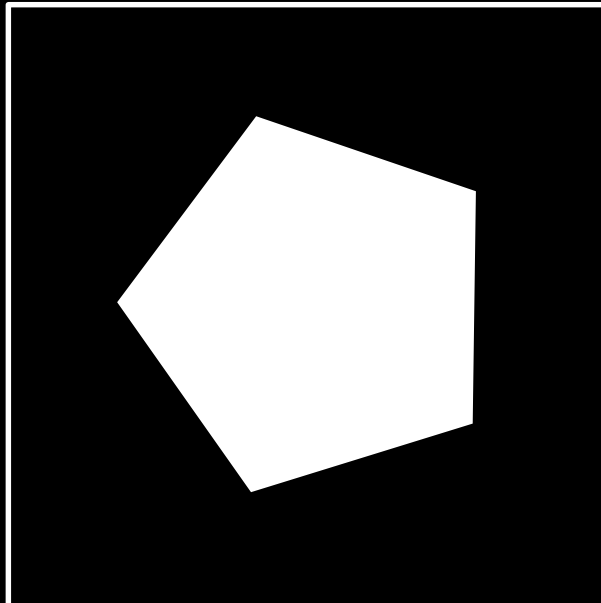
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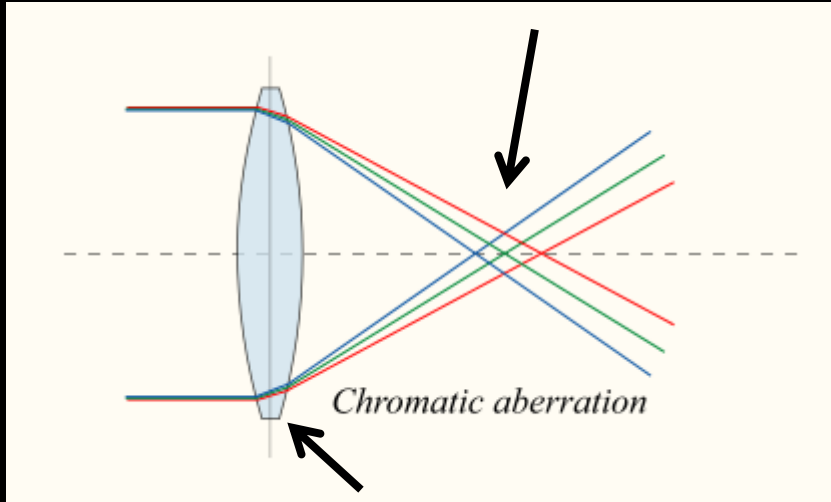
- Delay all plane waves so that they have the same phase at focal point
- The aperture of a real lens creates additional diffraction

Diffraction in lenses



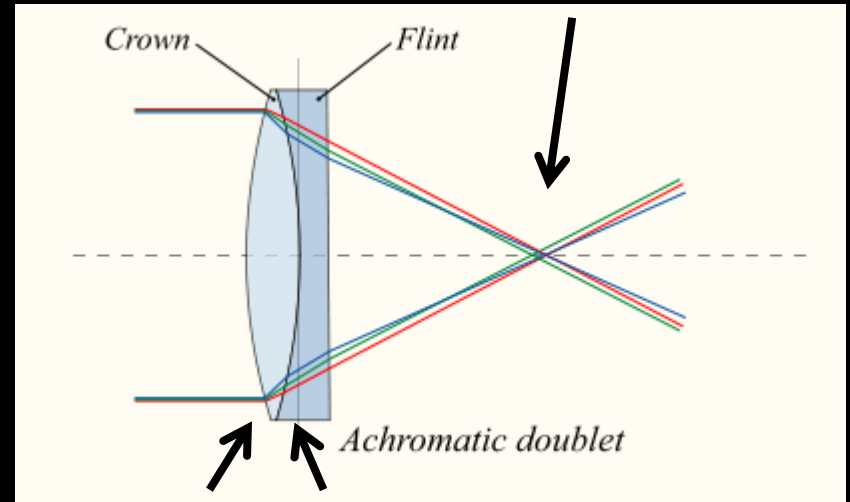
Chromatic aberration

focal length shifts
with wavelength



glass has dispersion
(refractive index changes
with wavelength)

one lens cancels out
dispersion of other

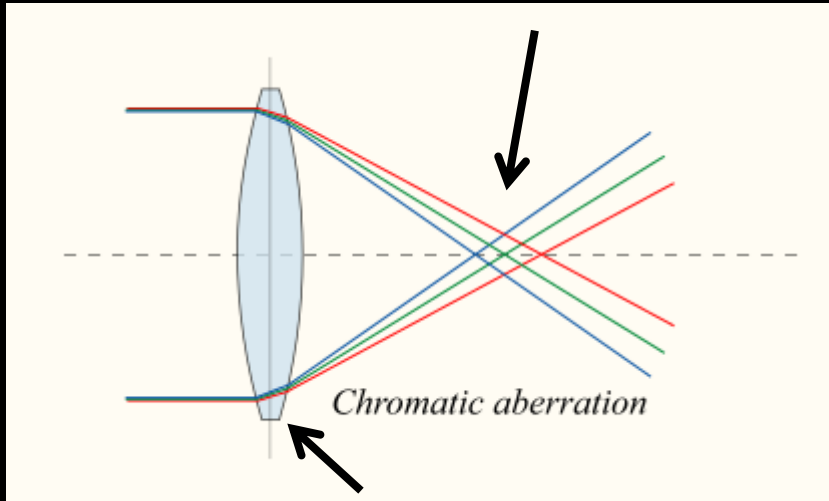


glasses of different
refractive index

How does Fraunhofer diffraction explain chromatic aberration?

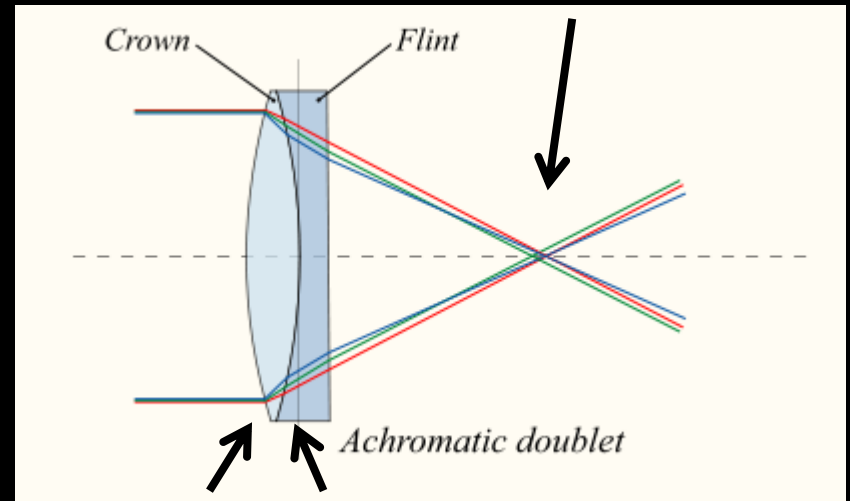
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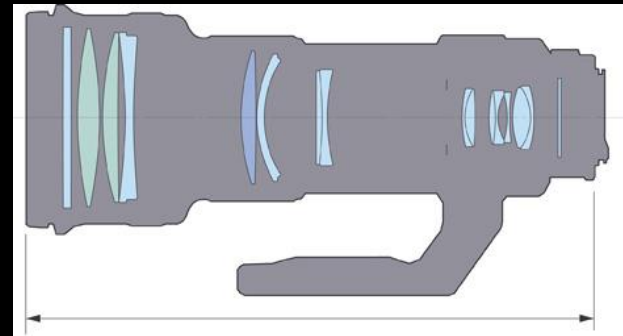
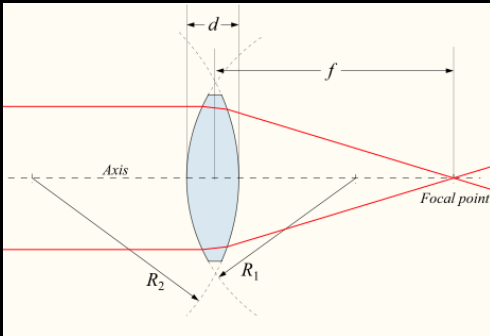
glasses of different
refractive index

How does Fraunhofer diffraction explain chromatic aberration?

- All our Fourier transforms
are wavelength-dependent

$$P(k) = \text{Fourier}\{p(r)\}$$
$$\Psi(k) = \text{Fourier}\{\psi(r)\}$$
$$k = \frac{2\pi c}{\lambda} n$$

Good “thin” lenses are compound lenses



dreaded camera bulge

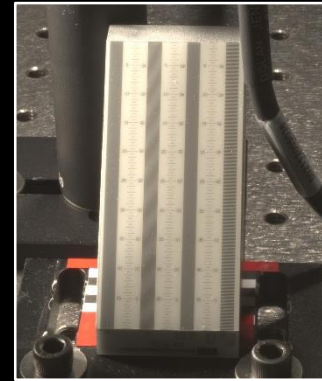
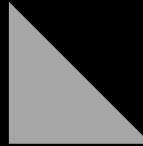


A small demonstration

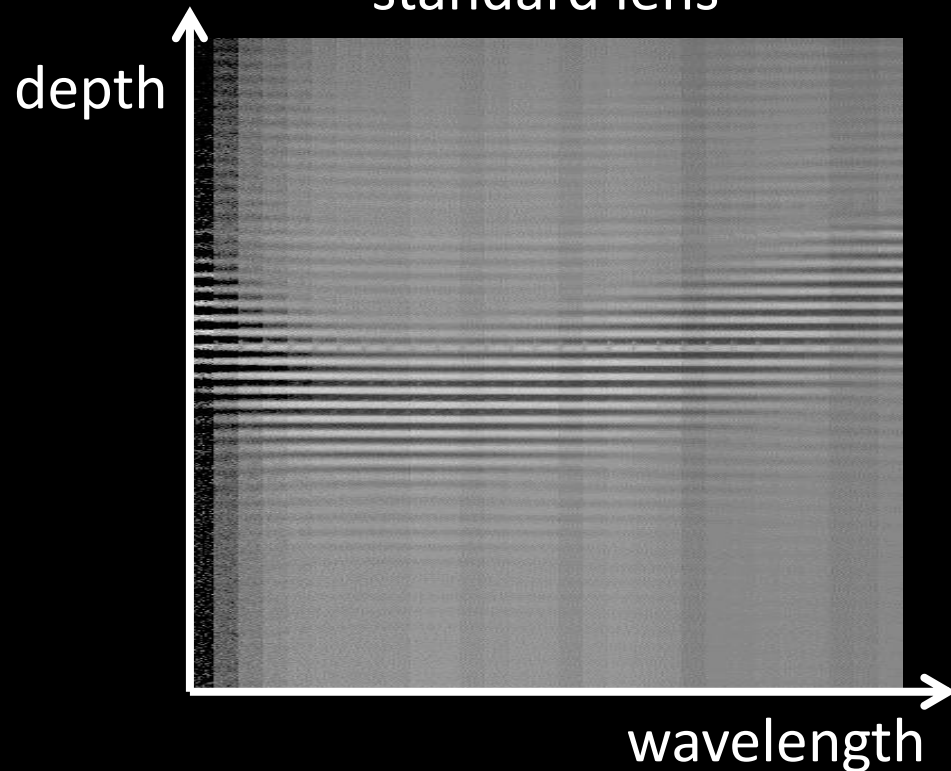
hyperspectral camera



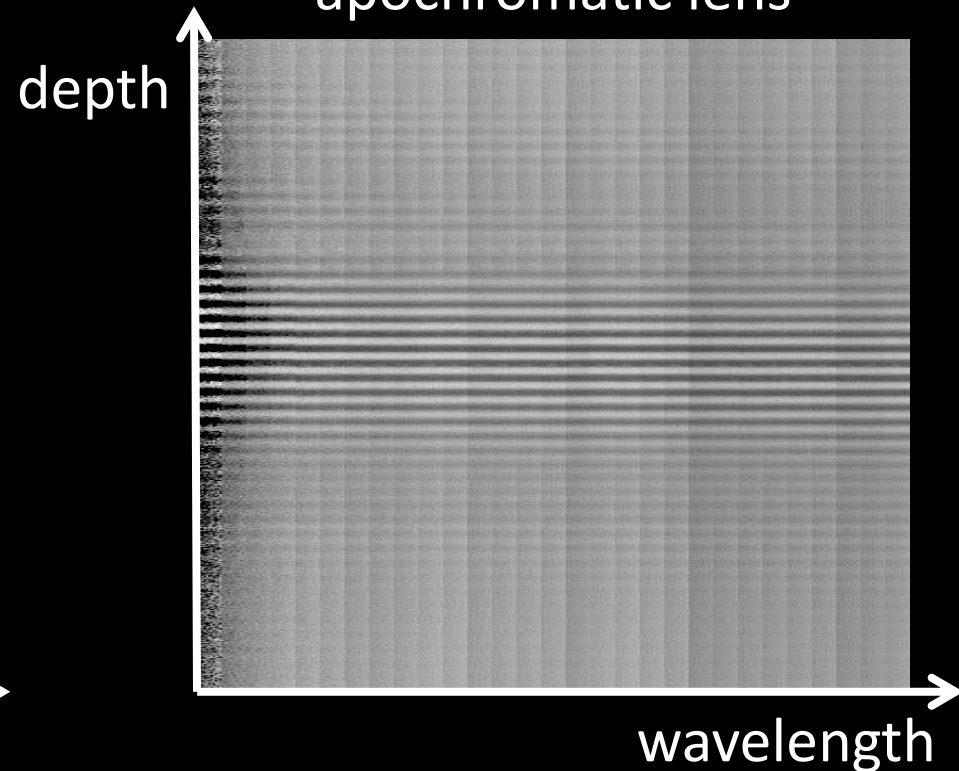
depth-of-field target



standard lens

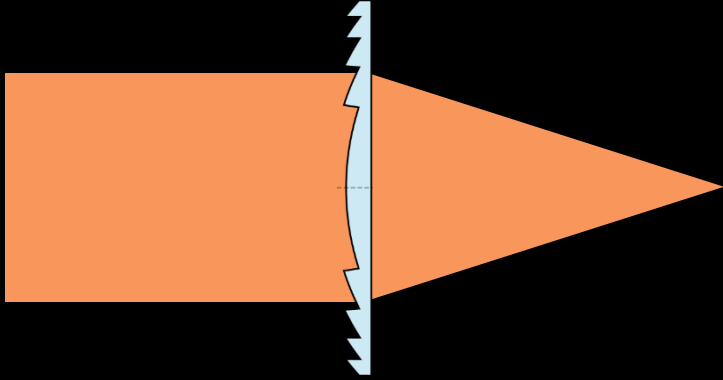


apochromatic lens

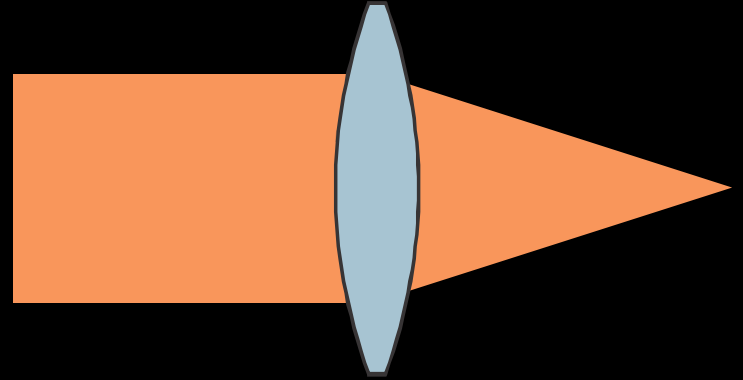


Fresnel lenses

also called diffractive lenses



- operation based on diffraction
- width stays roughly constant with aperture size



- width scales roughly linearly with aperture size

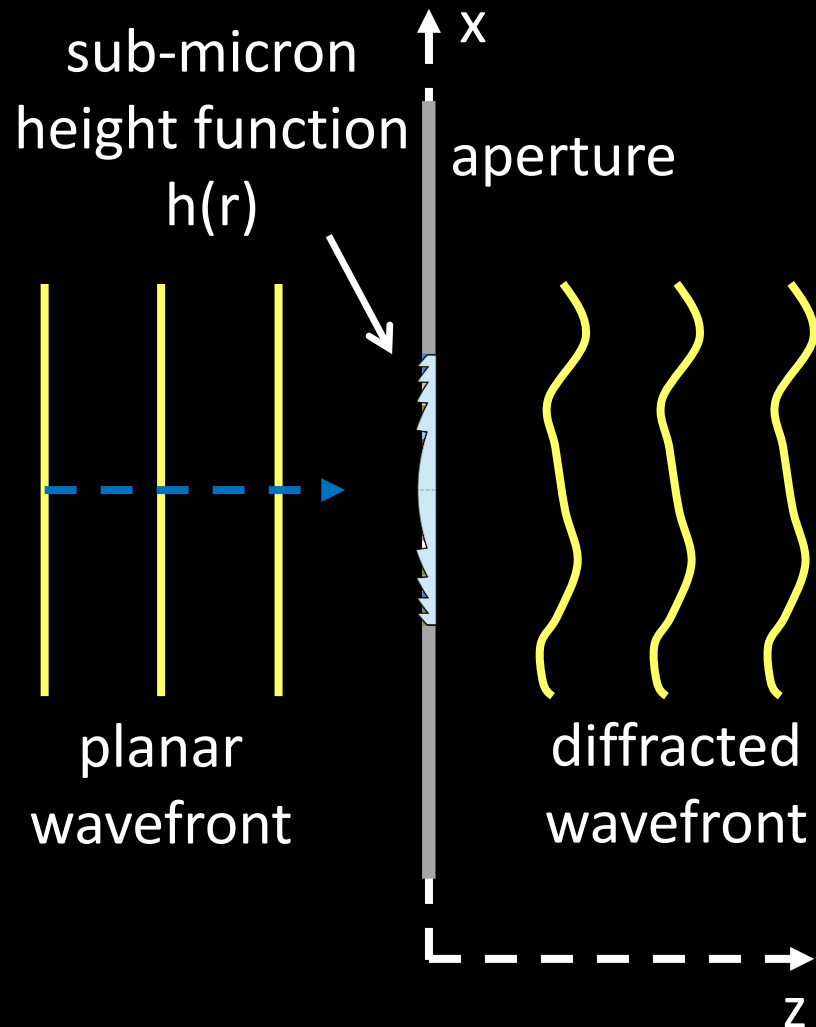


Fresnel lenses

solar grill



Fresnel lens



- transmission function:

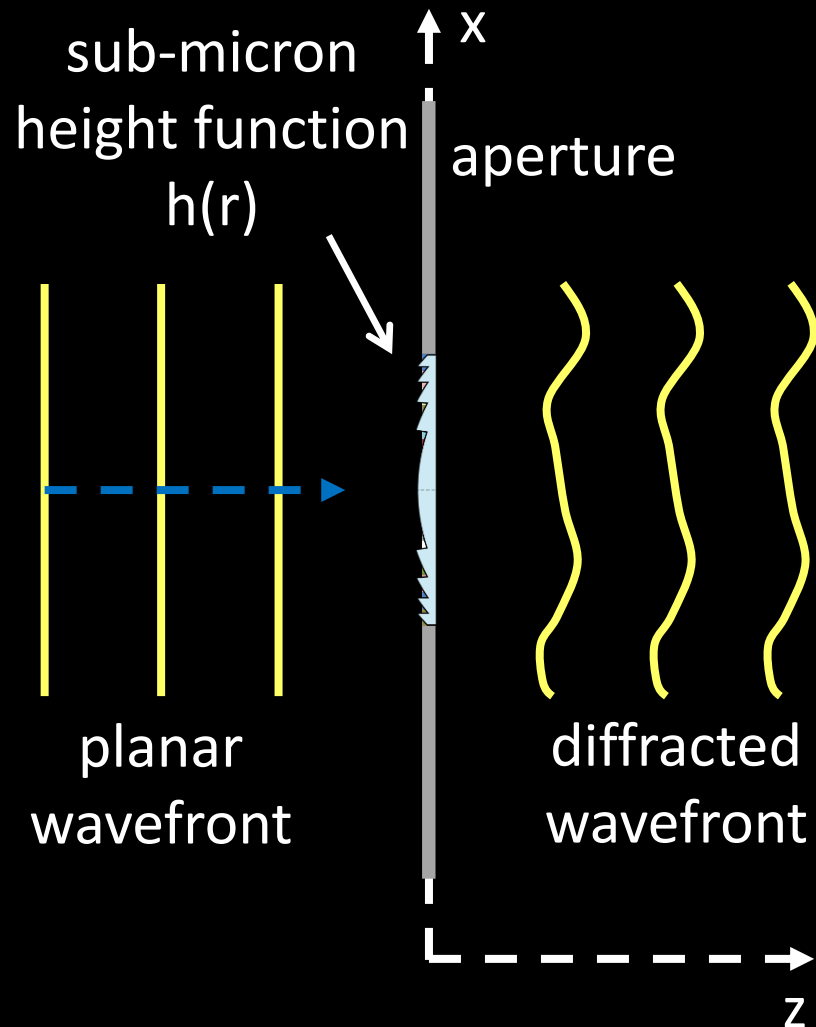
$$p(r) = A(r) \cdot \exp(j \cdot \Phi(r))$$

$$A(r) = \text{const}, \Phi(r) = c(\lambda) \cdot h(r)$$

Like a standard lens:

- Phase-only modulation.
- Delay all plane waves so that they have the same phase at focal point.

Fresnel lens



- transmission function:

$$p(r) = A(r) \cdot \exp(j \cdot \Phi(r))$$

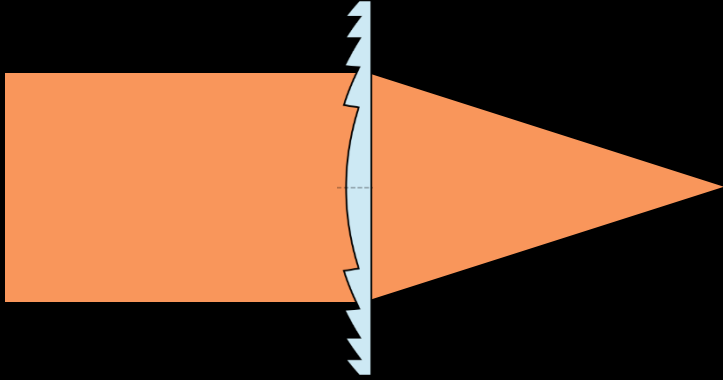
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Like a standard lens:

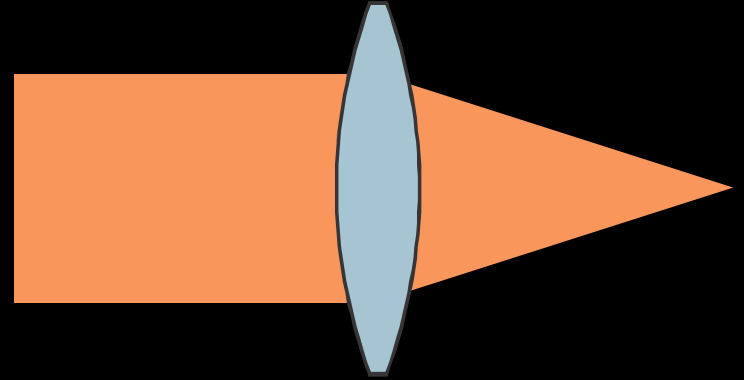
- Phase-only modulation.
- Delay all plane waves so that they have the same phase at focal point.

Fresnel lenses

also called diffractive lenses



- width stays roughly constant with aperture size



- width scales roughly linearly with aperture size

✓ very thin

✗ very dispersing

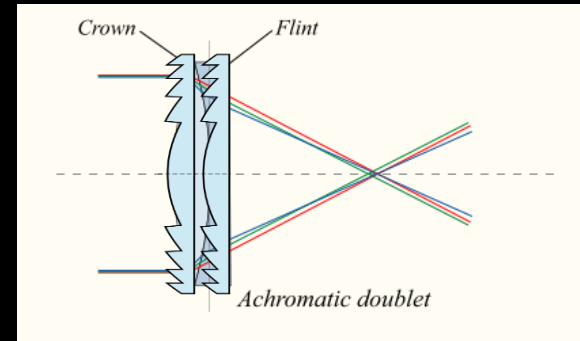
Diffraction achromat

conventional approach:

- multiple layers canceling out each other's aberration
- same principle as achromatic compound lens



bulky design
(thick and heavy)

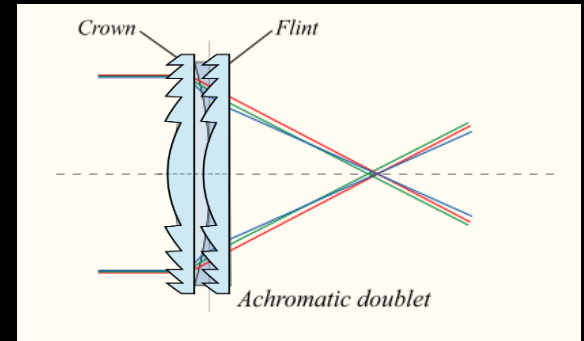


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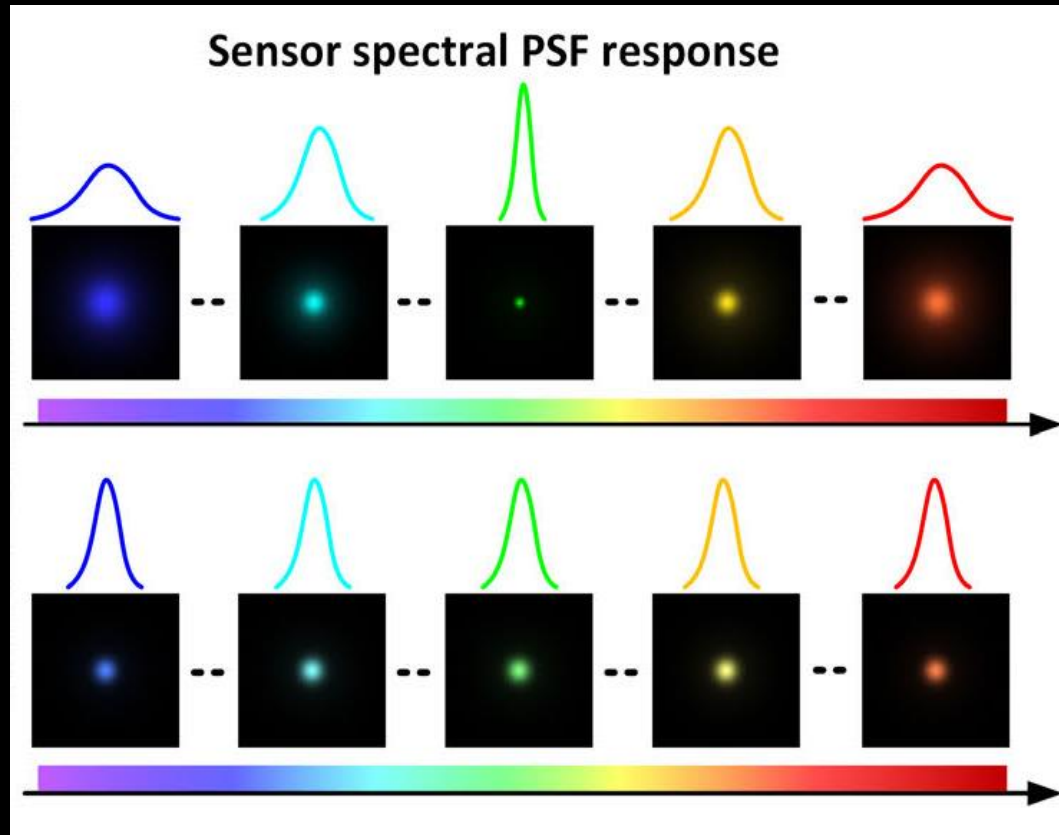


computational imaging approach:

- design single layer that has aberration that can be easily undone **computationally**
- possible because Fresnel lenses offer a lot more design flexibility (arbitrary height function)

Diffraction achromat

regular
Fresnel lens



sharp PSF at
center
wavelength

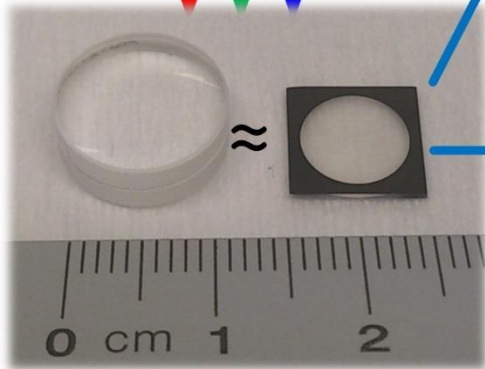
achromatic
Fresnel lens

blurry but
same PSF at all
wavelengths

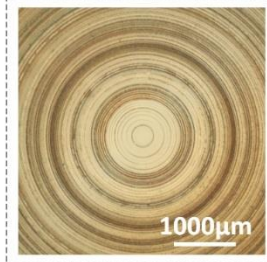
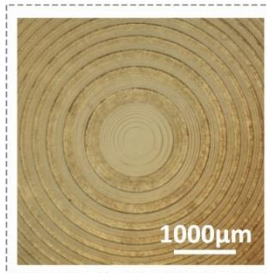
- Instead of making one wavelength sharp, make all of them equally blurred
- Fix aberration using non-blind deconvolution with same kernel for all wavelengths

Diffraction achromat

Full visible spectrum
illumination

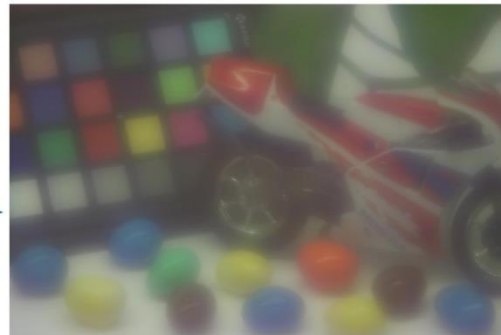
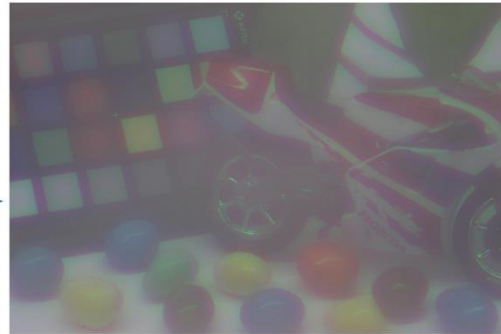


Fresnel lens



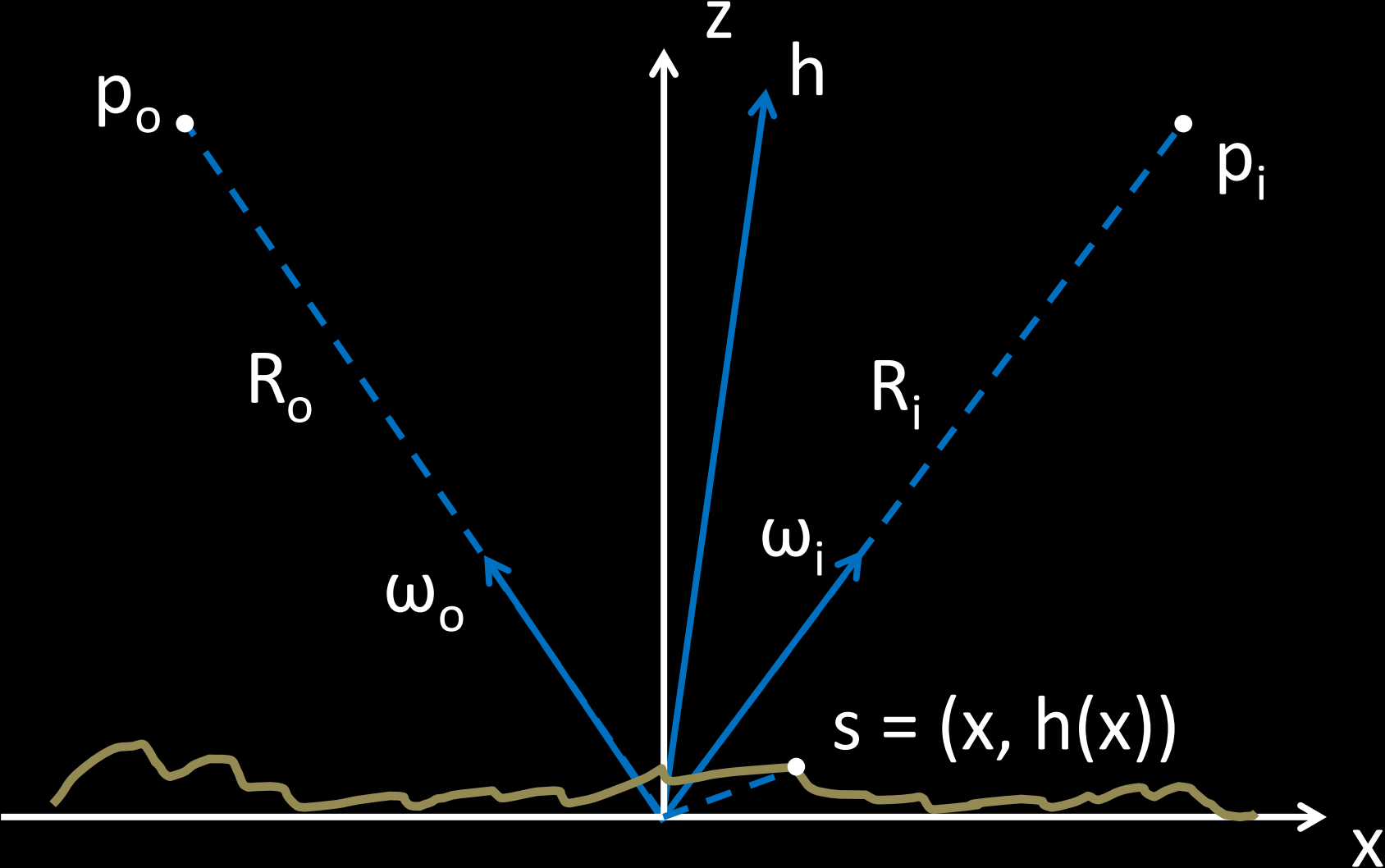
Diffraction
Achromat

Post-capture deconvolution



Fraunhofer diffraction and reflection

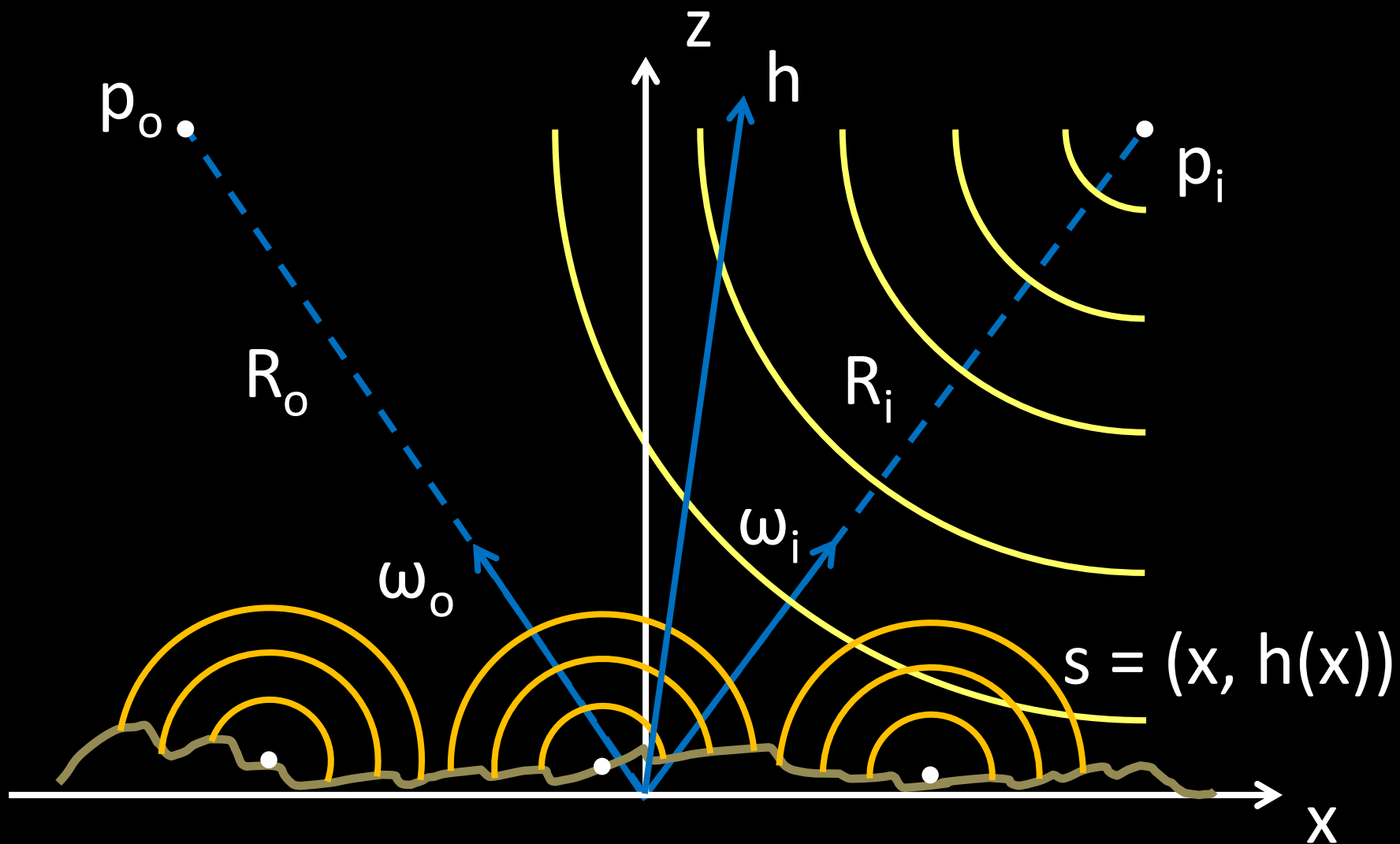
Bidirectional Reflectance Distribution Function (BRDF)



Setting

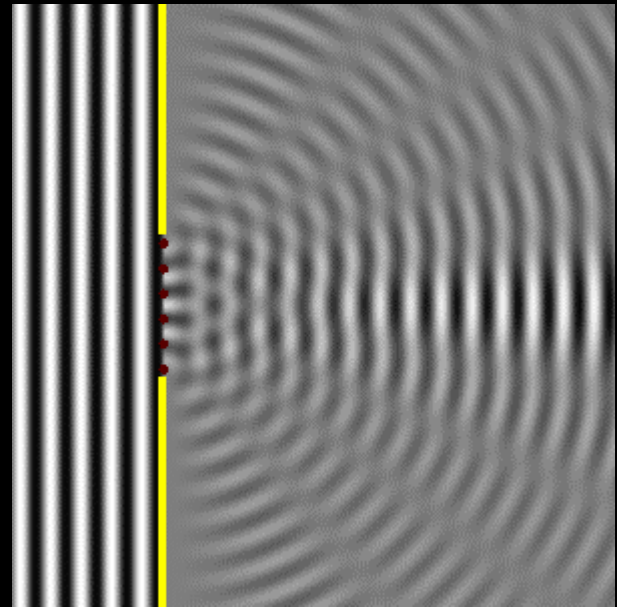
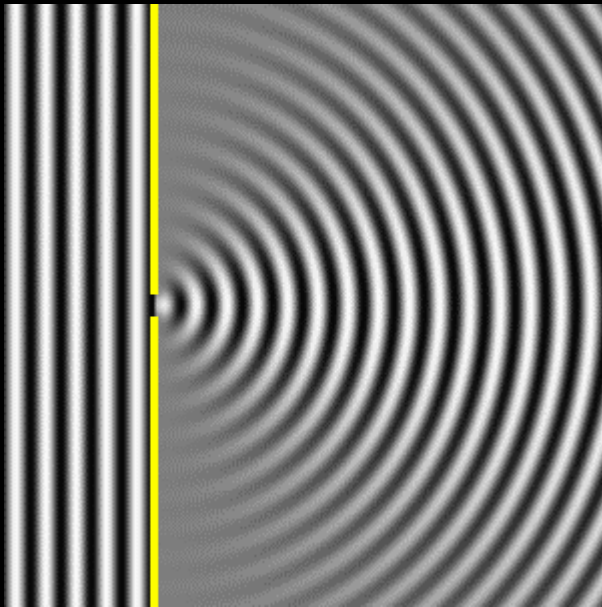
Huygen's principle

coherent illumination

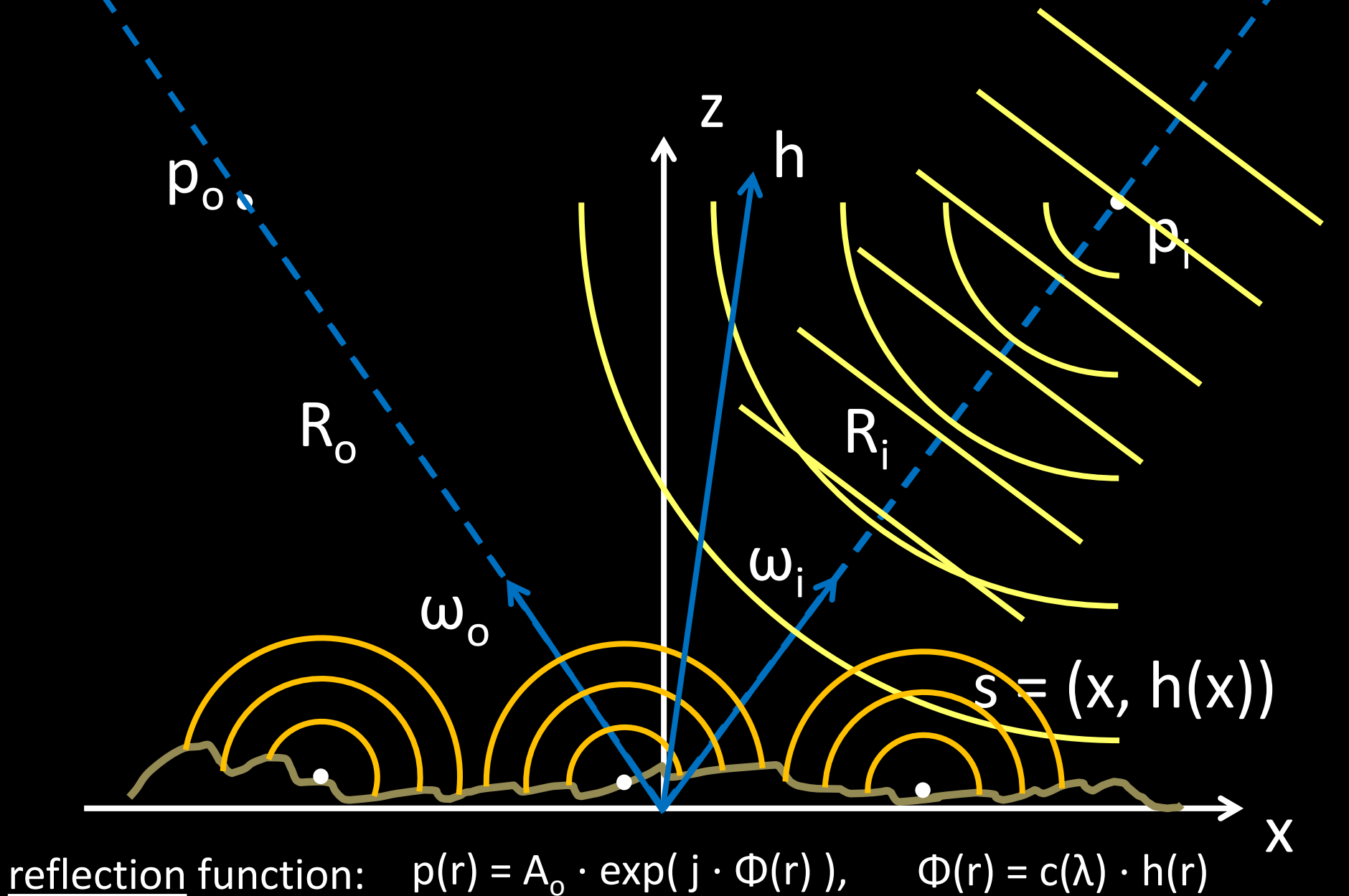


Huygen's principle

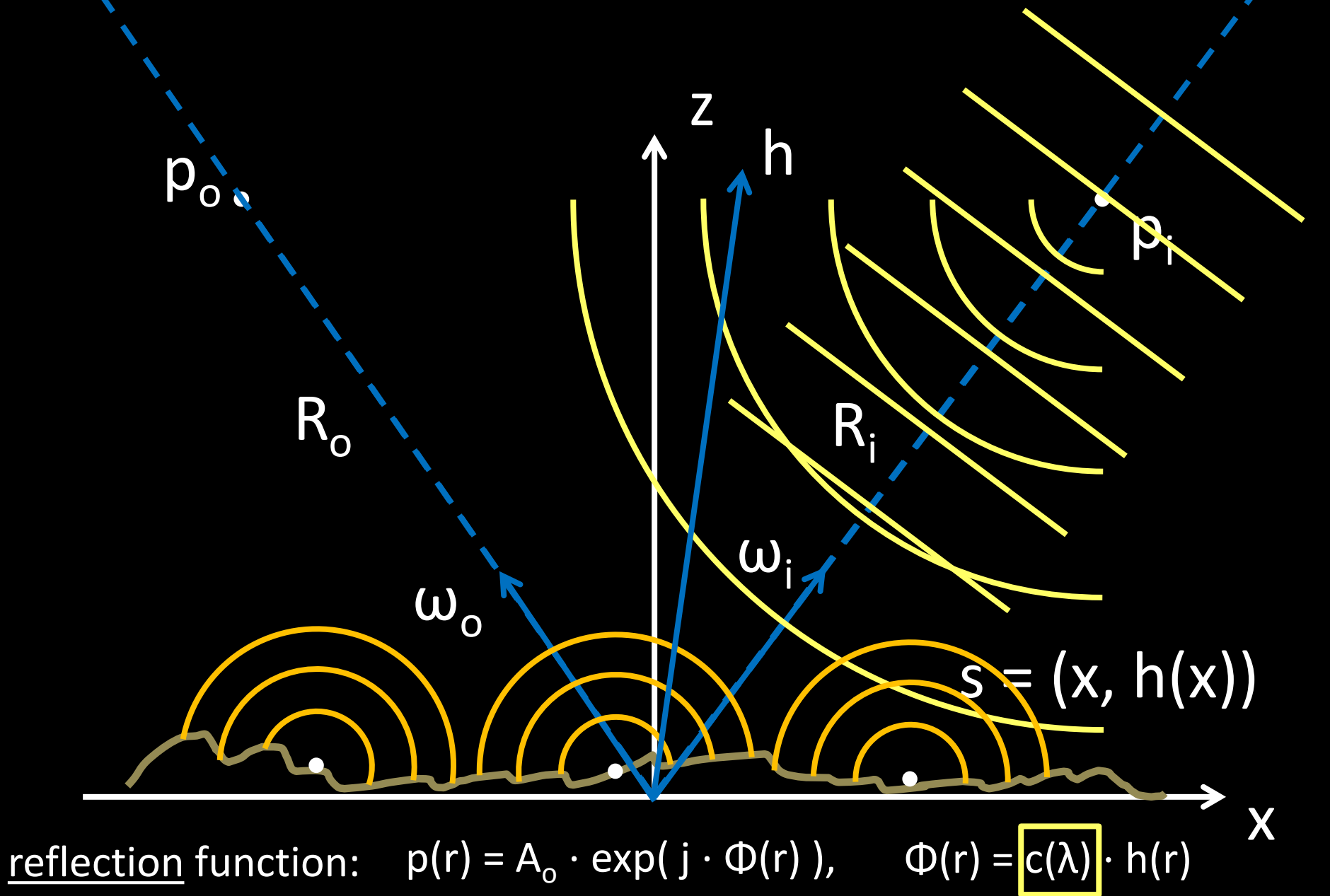
Under far-field approximation, it's equivalent to Fraunhofer diffraction



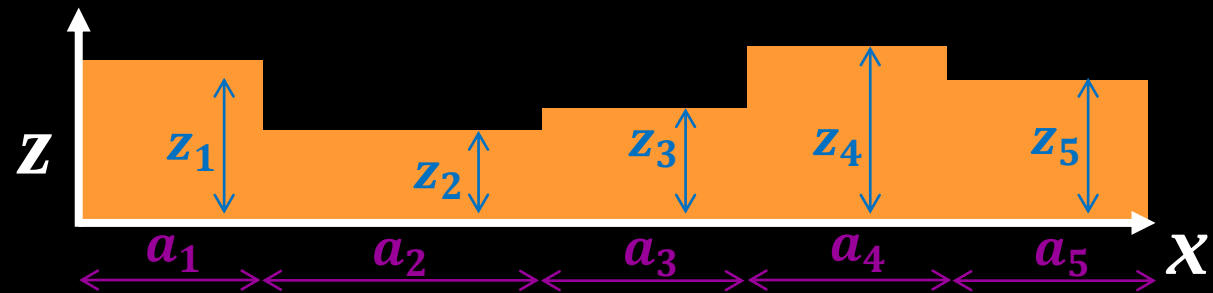
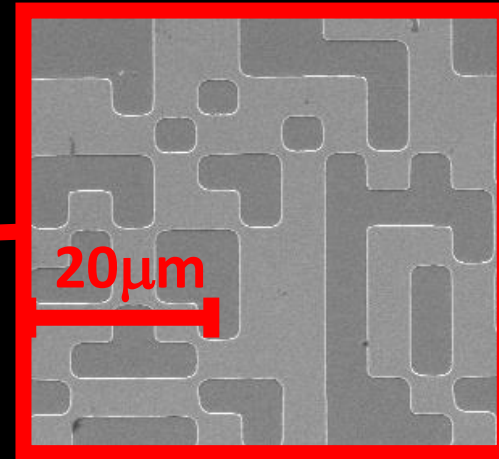
Far-field setting



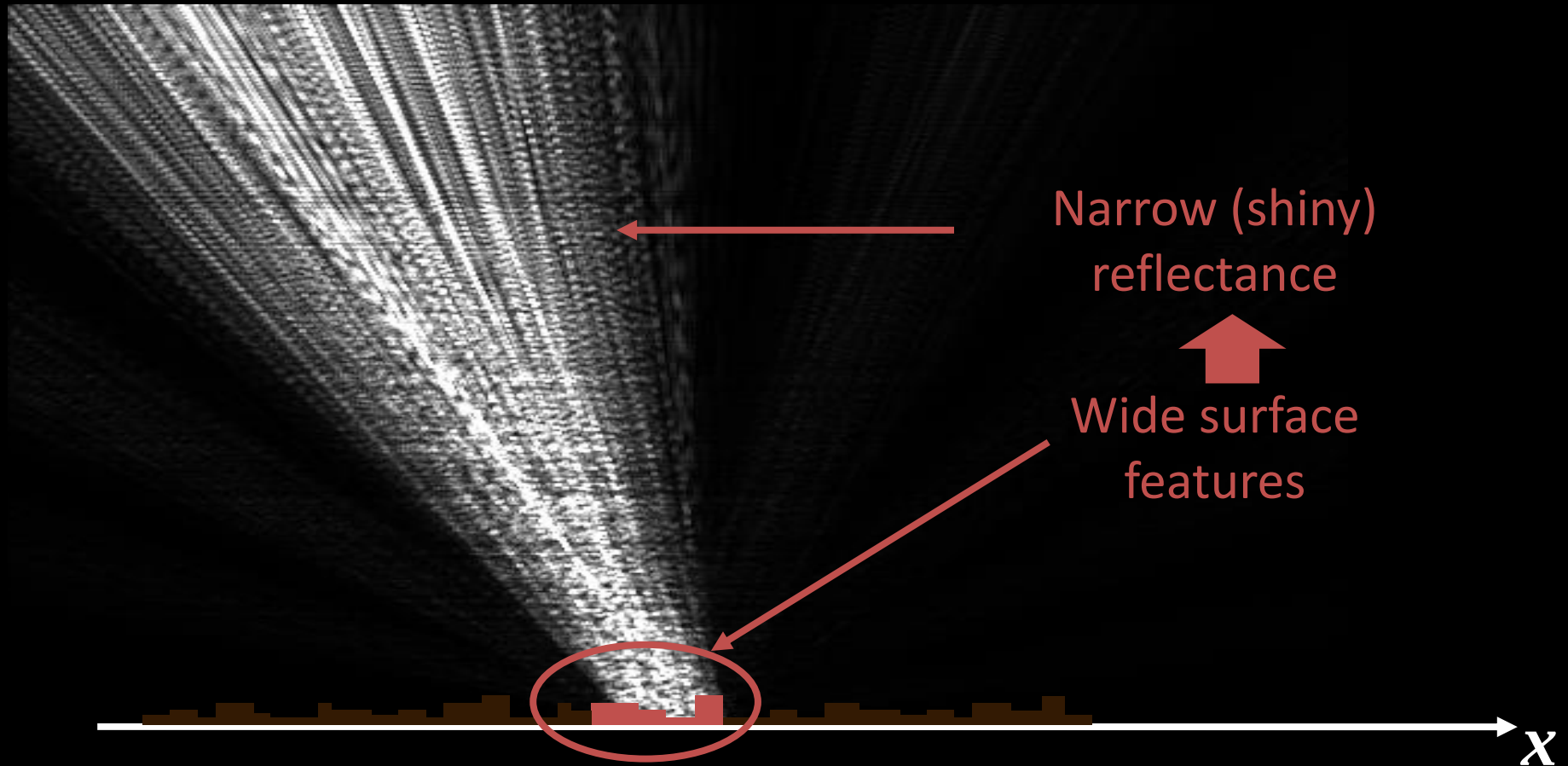
Far-field setting



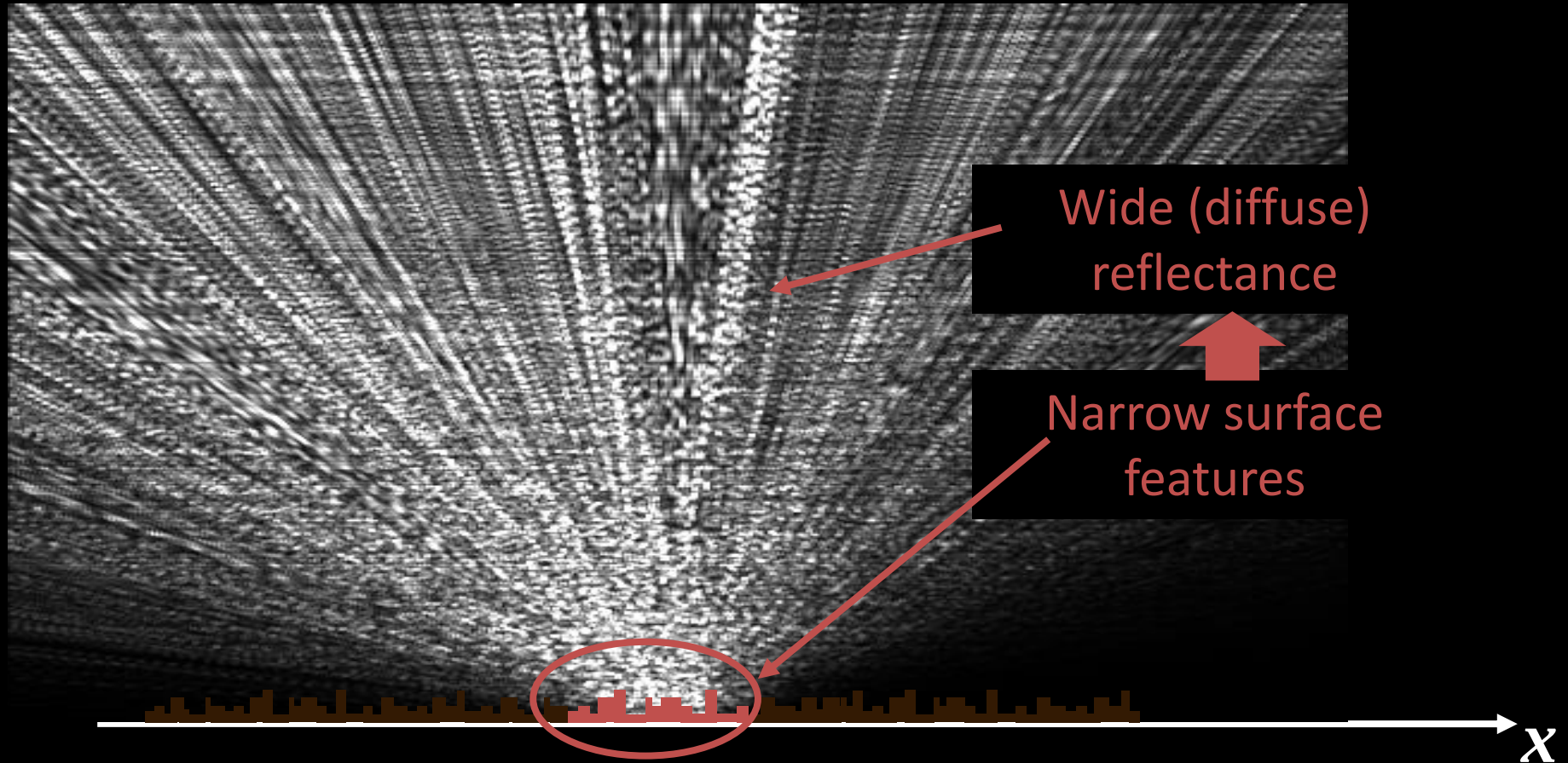
Photolithography



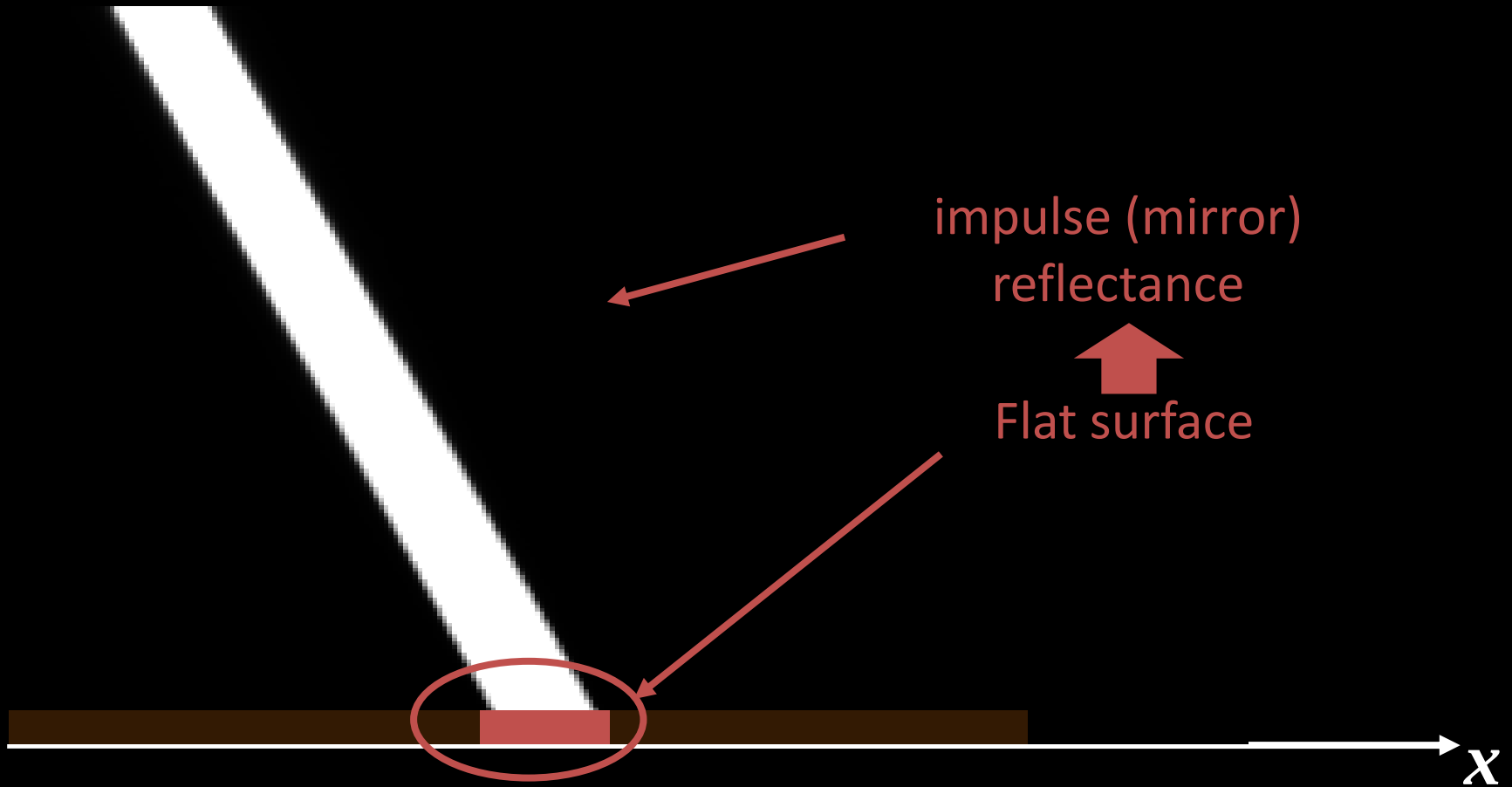
Inverse width relationship



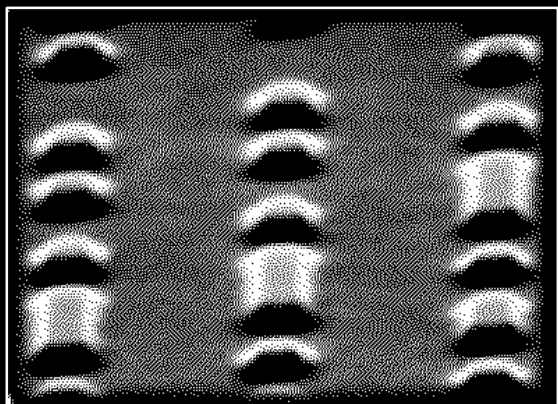
Inverse width relationship



Inverse width relationship



Diffractive BRDF renderings



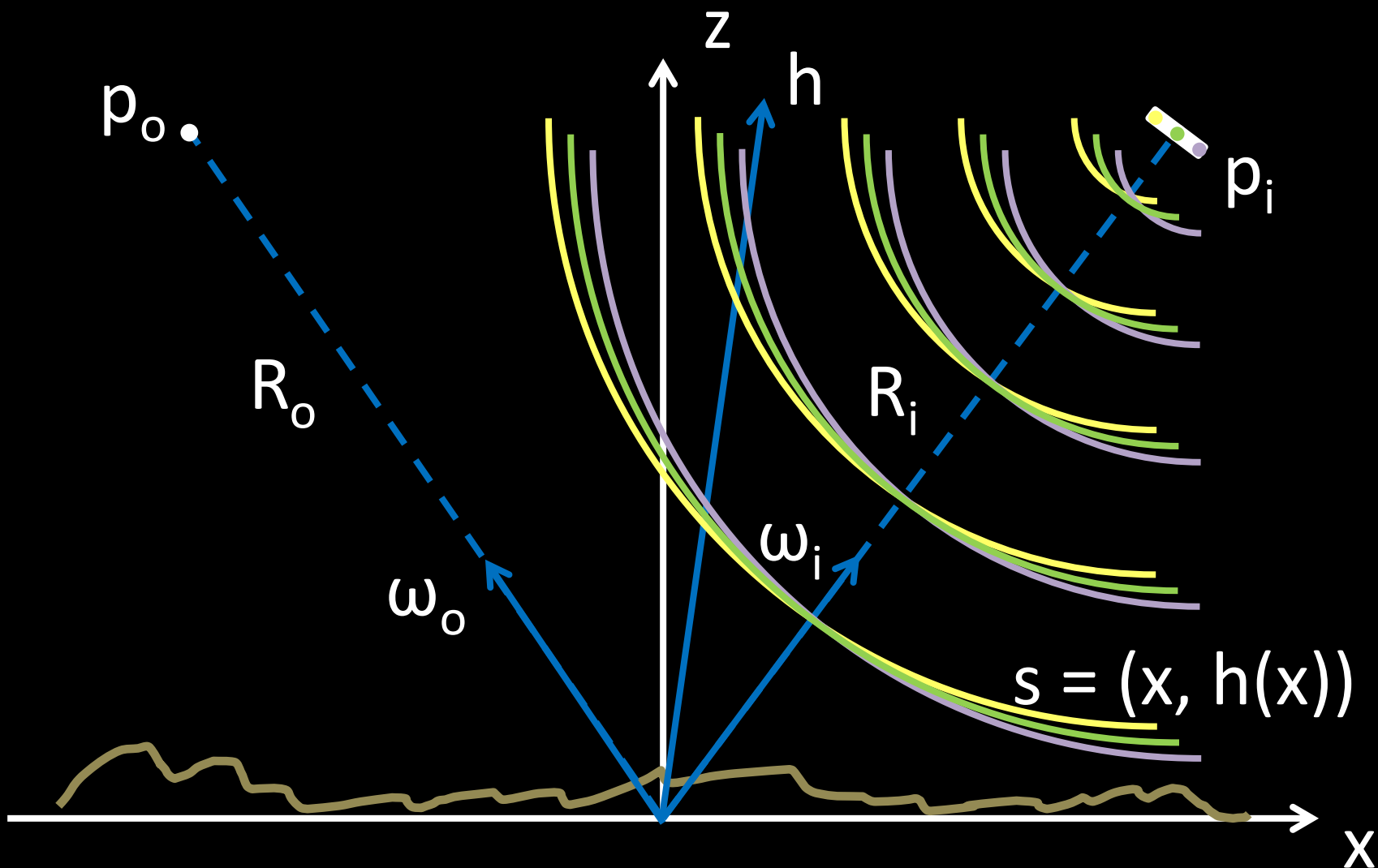
close-up of CD surface



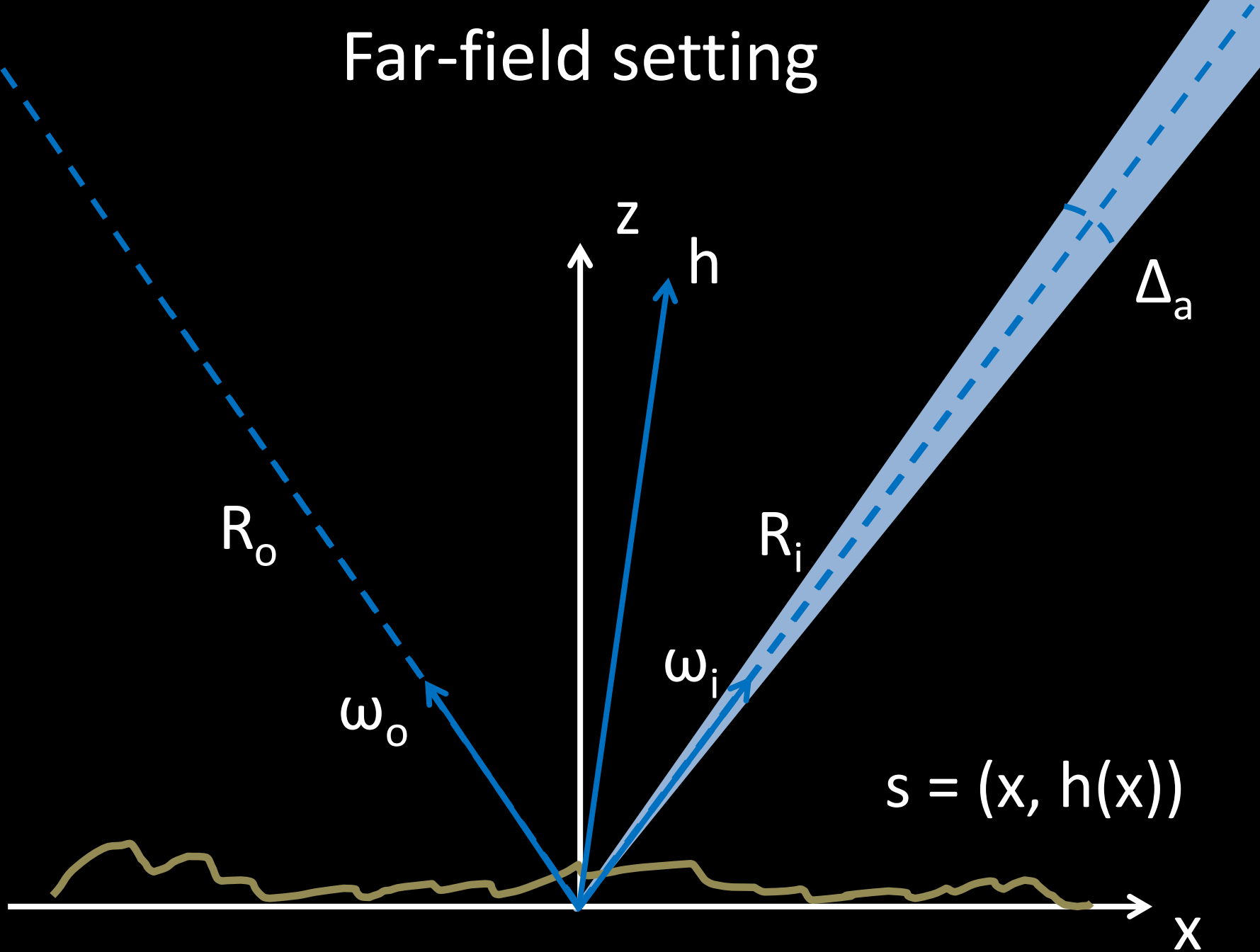
rendering

Setting

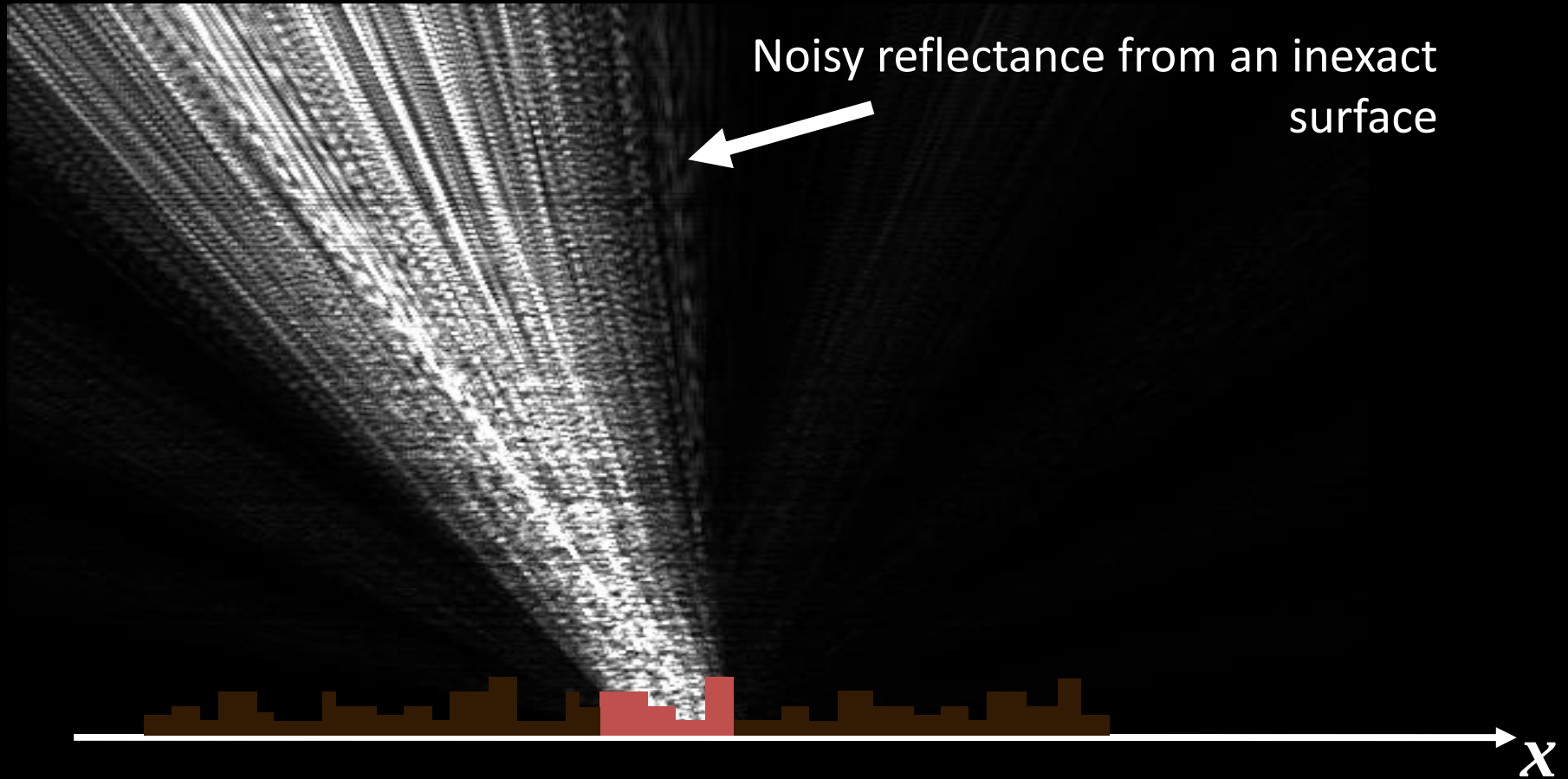
(spatially) incoherent illumination



Far-field setting

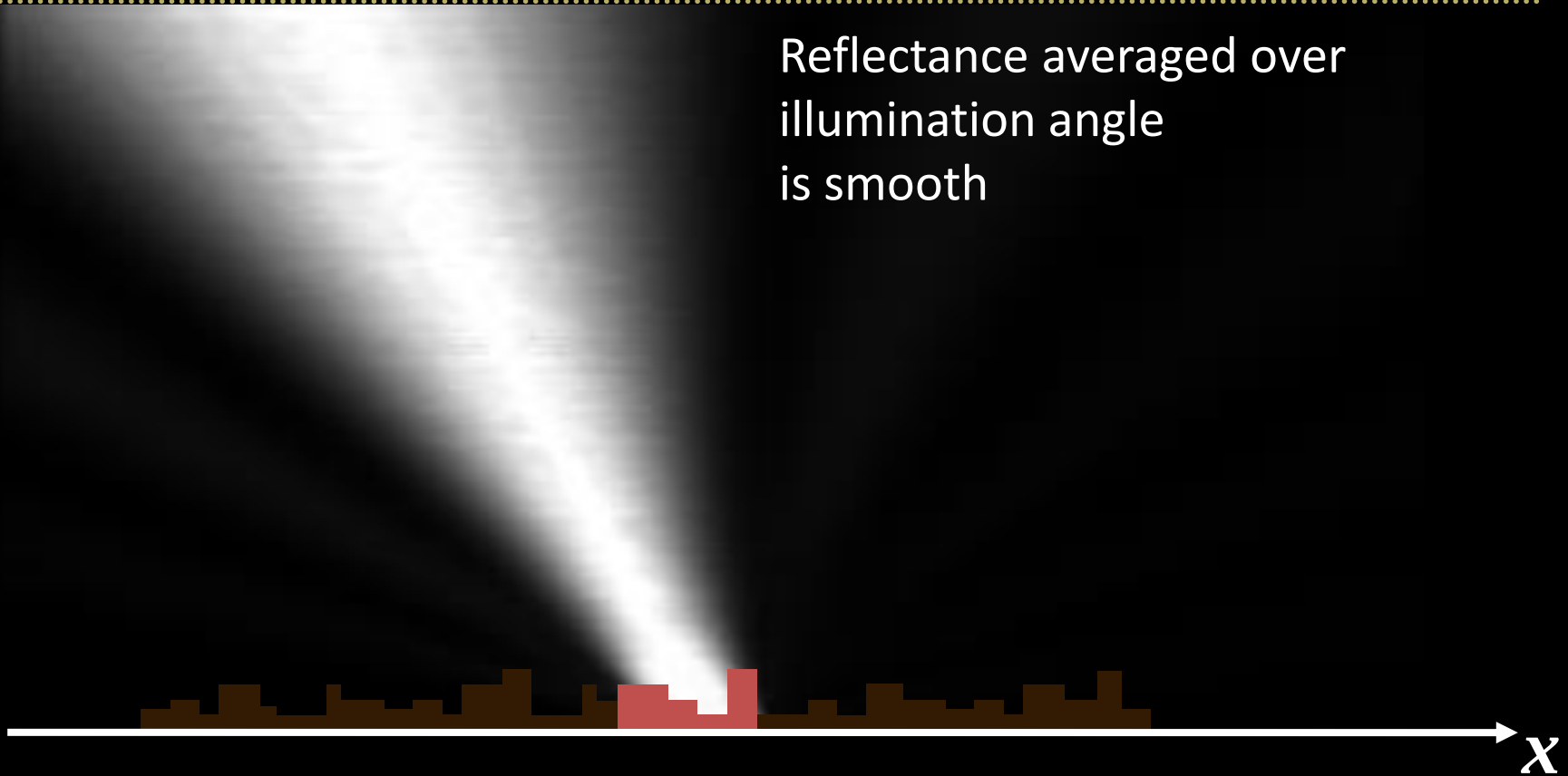


Speckles



Incoherent reflectance: blurring coherent reflectance by source angle

Reflectance averaged over
illumination angle
is smooth



**Fabricating BRDFs
at High Spatial Resolution
using Wave Optics**

**Anat Levin, Daniel Glasner, Ying Xiong,
Fredo Durand, William Freeman,
Wojciech Matusik and Todd Zickler.**

References

Basic reading:

- Goodman, “Introduction to Fourier Optics,” W. H. Freeman 2004.
This comprehensive textbook is the standard reference when it comes to Fourier optics.
- Peng et al., “The Diffractive Achromat: Full Spectrum Computational Imaging with Diffractive Optics,” SIGGRAPH 2016.
This paper discusses Fresnel lenses and how to use computational imaging to deal with chromatic aberration.
- Stam, “Diffractive shaders,” SIGGRAPH 1999.
- Levin et al., “Fabricating BRDFs at high spatial resolution using wave optics,” SIGGRAPH 2013
These two papers discuss Fraunhofer diffraction for the reflective case.

Additional reading:

- Glasner et al., “A Reflectance Display,” SIGGRAPH 2014.
- Ye et al., “Toward BxDF Display using Multilayer Diffraction,” SIGGRAPH Asia 2014.
- Levin et al., “Passive light and viewpoint sensitive display of 3D content,” ICCP 2016.
These three papers discuss how to use diffraction to build passive reactive displays.
- Damberg et al., “High Brightness HDR Projection Using Dynamic Freeform Lensing,” TOG 2016
This paper discusses how to use diffraction to create lenses of arbitrary focusing patterns.
- Zhang and Levoy, “Wigner Distributions and How They Relate to the Light Field”, ICCP 2009.
- Cuyper et al., “Reflectance Model for Diffraction”, TOG 2012.
These two papers discuss the relationship between Fourier optics, ray optics and lightfields, and the Wigner transformation.