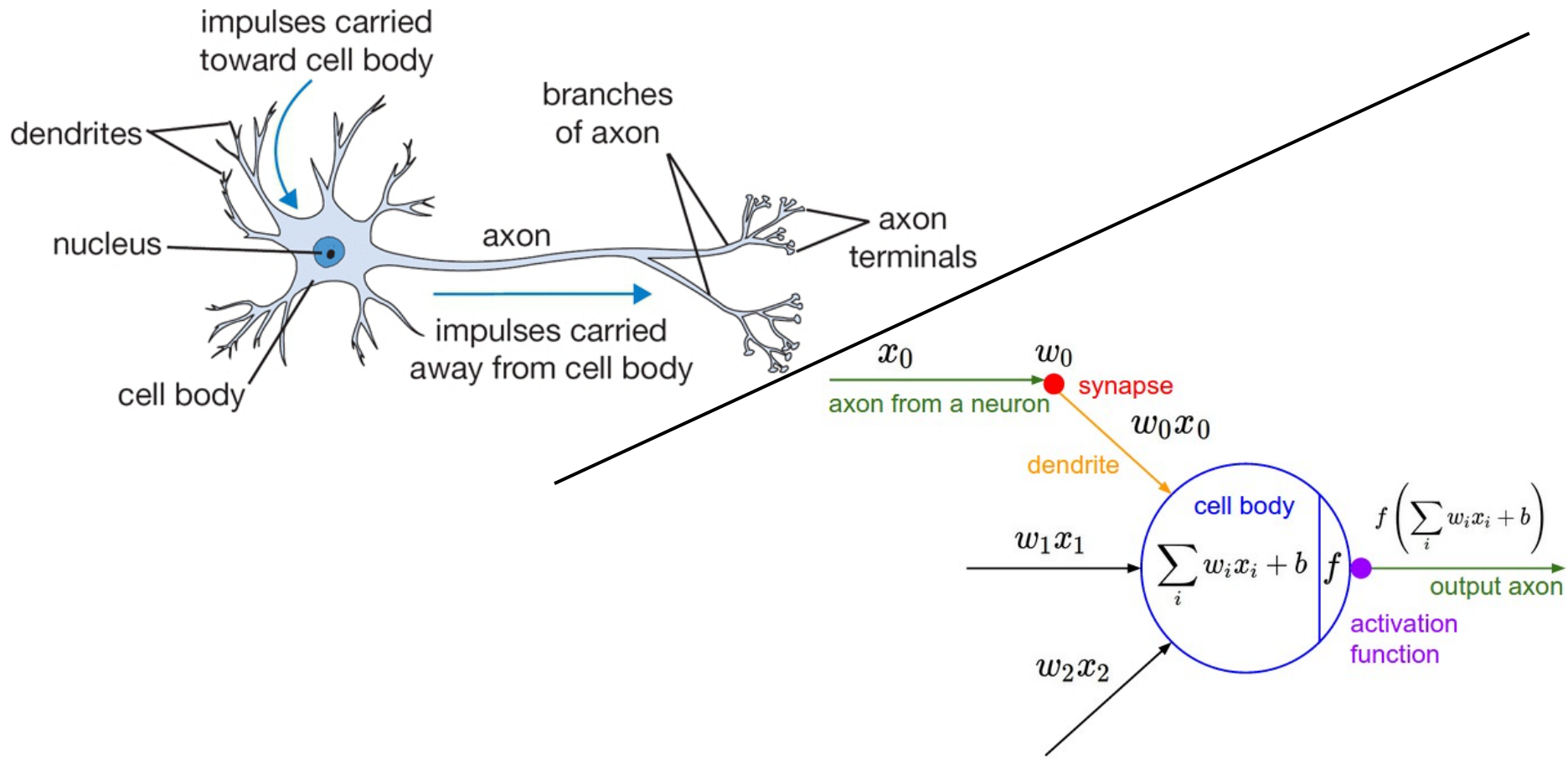


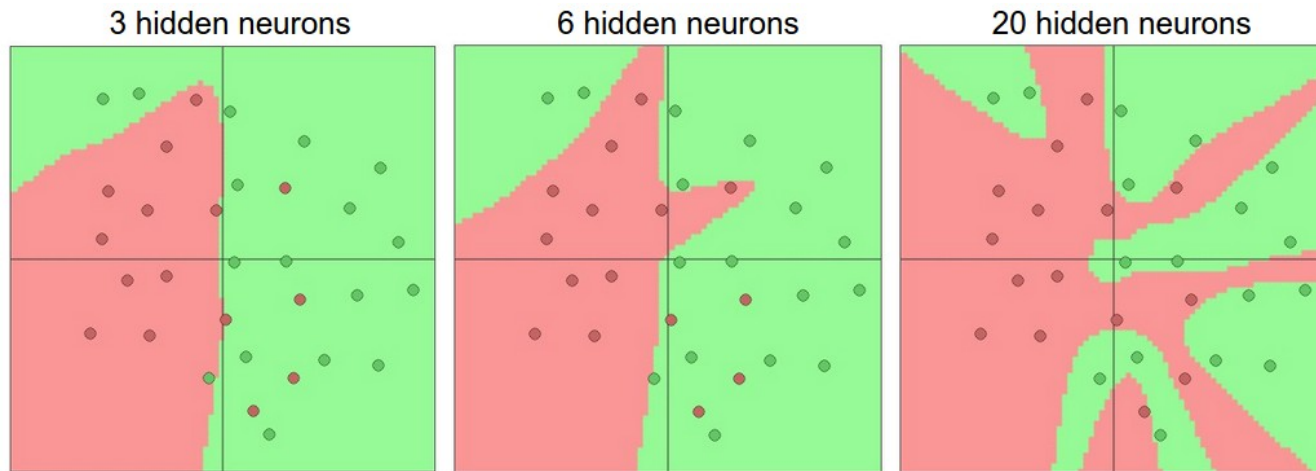
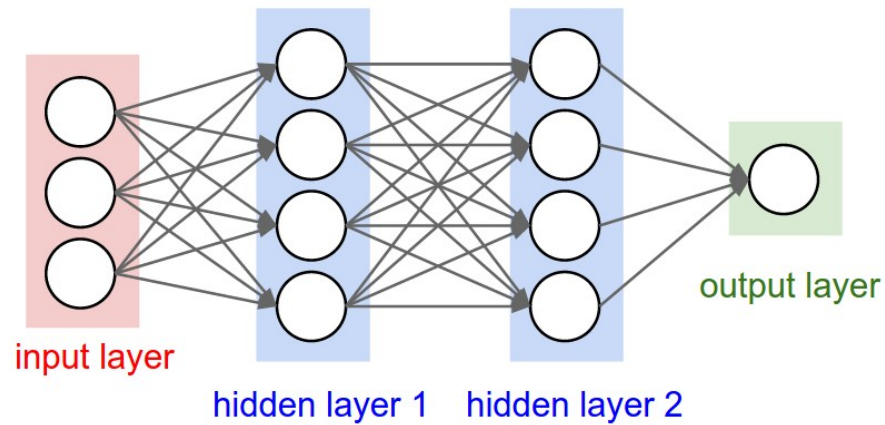
Administrative

- **A2** is out. It was late 2 days so due date will be shifted by ~2 days.
- we updated the project page with many pointers to datasets.

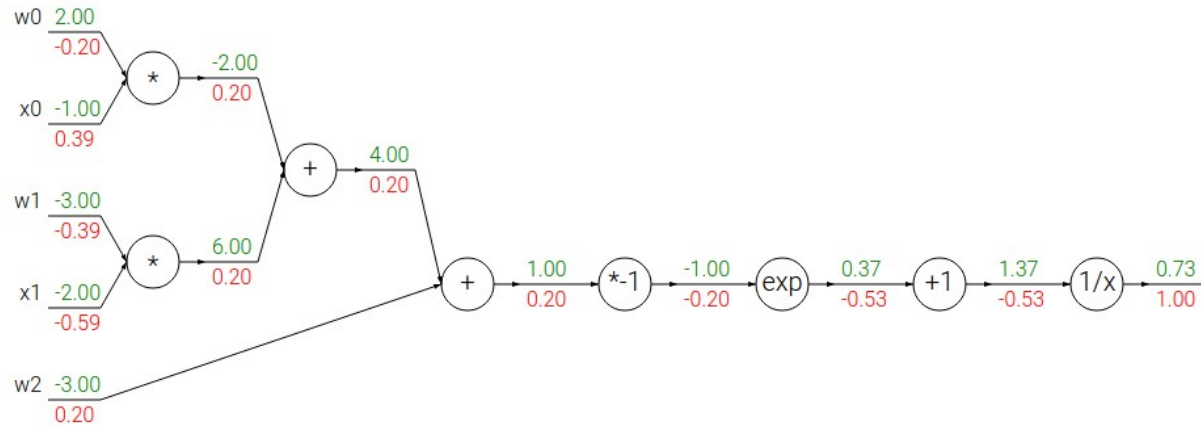
Lecture 6:

Training Neural Networks





Backpropagation (recursive chain rule)

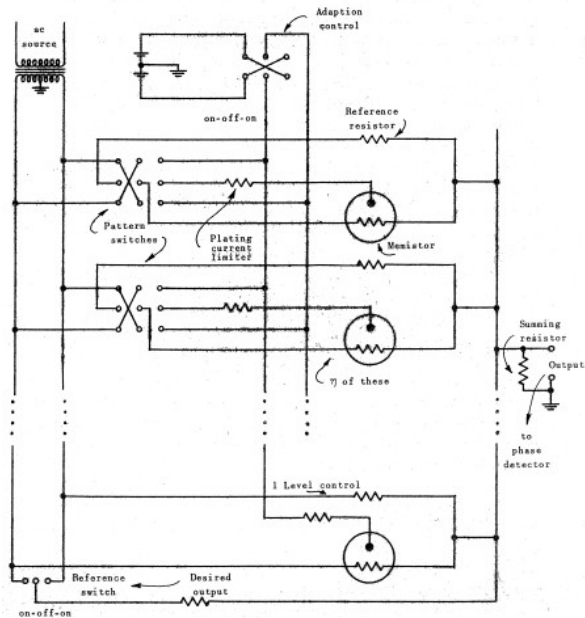
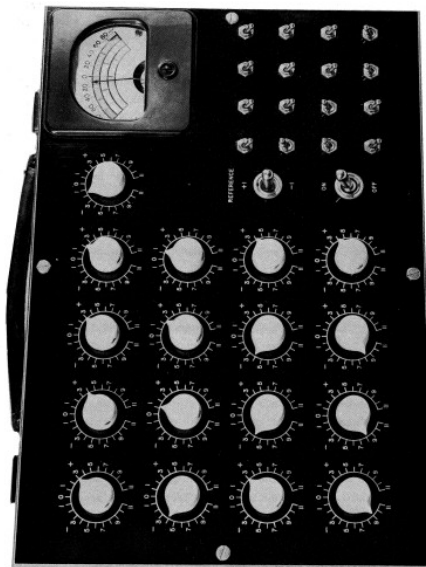
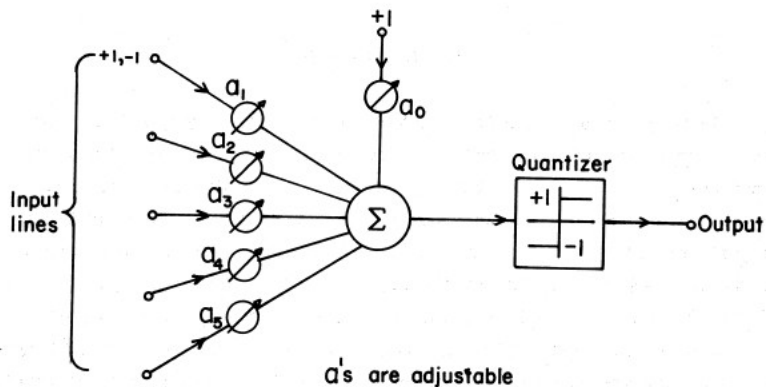


Mini-batch Gradient descent

Loop:

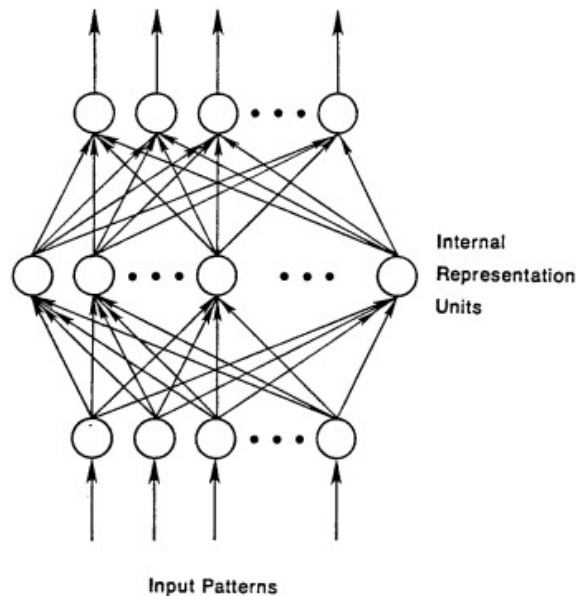
1. Sample a batch of data
2. Backprop to calculate the analytic gradient
3. Perform a parameter update

A bit of history



Widrow and Hoff, ~1960: Adaline

A bit of history



To be more specific, then, let

$$E_p = \frac{1}{2} \sum_j (t_{pj} - o_{pj})^2 \quad (2)$$

be our measure of the error on input/output pattern p and let $E = \sum E_p$ be our overall measure of the error. We wish to show that the delta rule implements a gradient descent in E when the units are linear. We will proceed by simply showing that

$$-\frac{\partial E_p}{\partial w_{ji}} = \delta_{pj} i_{pi}$$

recognizable maths

which is proportional to $\Delta_p w_{ji}$ as prescribed by the delta rule. When there are no hidden units it is straightforward to compute the relevant derivative. For this purpose we use the chain rule to write the derivative as the product of two parts: the derivative of the error with respect to the output of the unit times the derivative of the output with respect to the weight.

$$\frac{\partial E_p}{\partial w_{ji}} = \frac{\partial E_p}{\partial o_{pj}} \frac{\partial o_{pj}}{\partial w_{ji}}. \quad (3)$$

The first part tells how the error changes with the output of the j th unit and the second part tells how much changing w_{ji} changes that output. Now, the derivatives are easy to compute. First, from Equation 2

$$\frac{\partial E_p}{\partial o_{pj}} = -(t_{pj} - o_{pj}) = -\delta_{pj}. \quad (4)$$

Not surprisingly, the contribution of unit u_j to the error is simply proportional to δ_{pj} . Moreover, since we have linear units,

$$o_{pj} = \sum_i w_{ji} i_{pi}, \quad (5)$$

from which we conclude that

$$\frac{\partial o_{pj}}{\partial w_{ji}} = i_{pi}.$$

Thus, substituting back into Equation 3, we see that

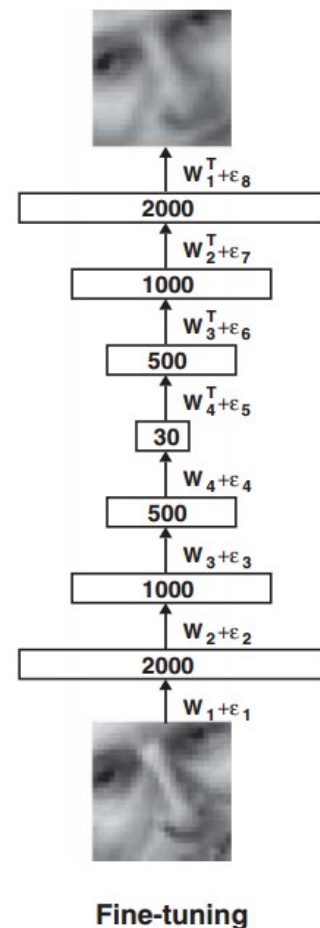
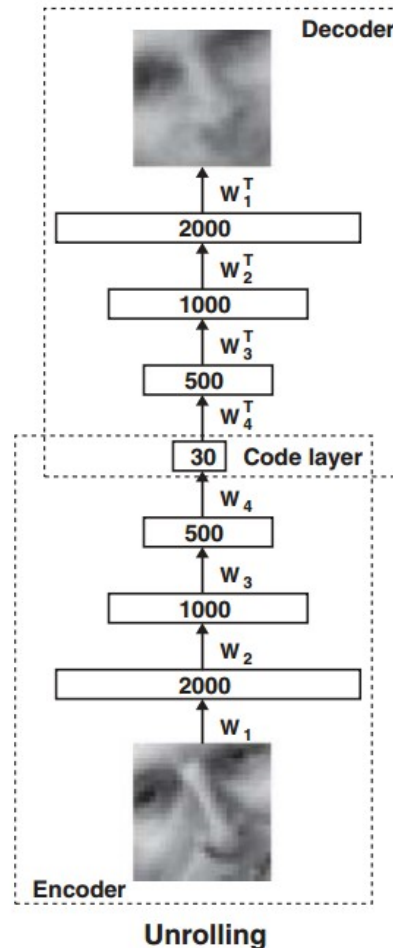
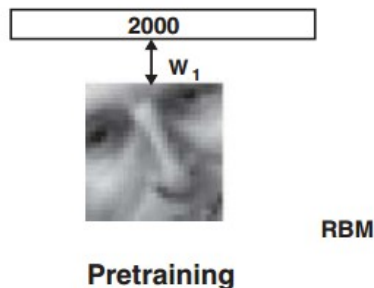
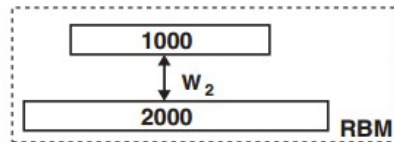
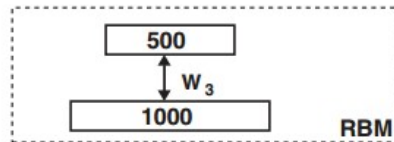
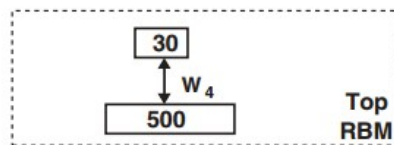
$$-\frac{\partial E_p}{\partial w_{ji}} = \delta_{pj} i_{pi} \quad (6)$$

Rumelhart et al. 1986: First time back-propagation became popular

A bit of history

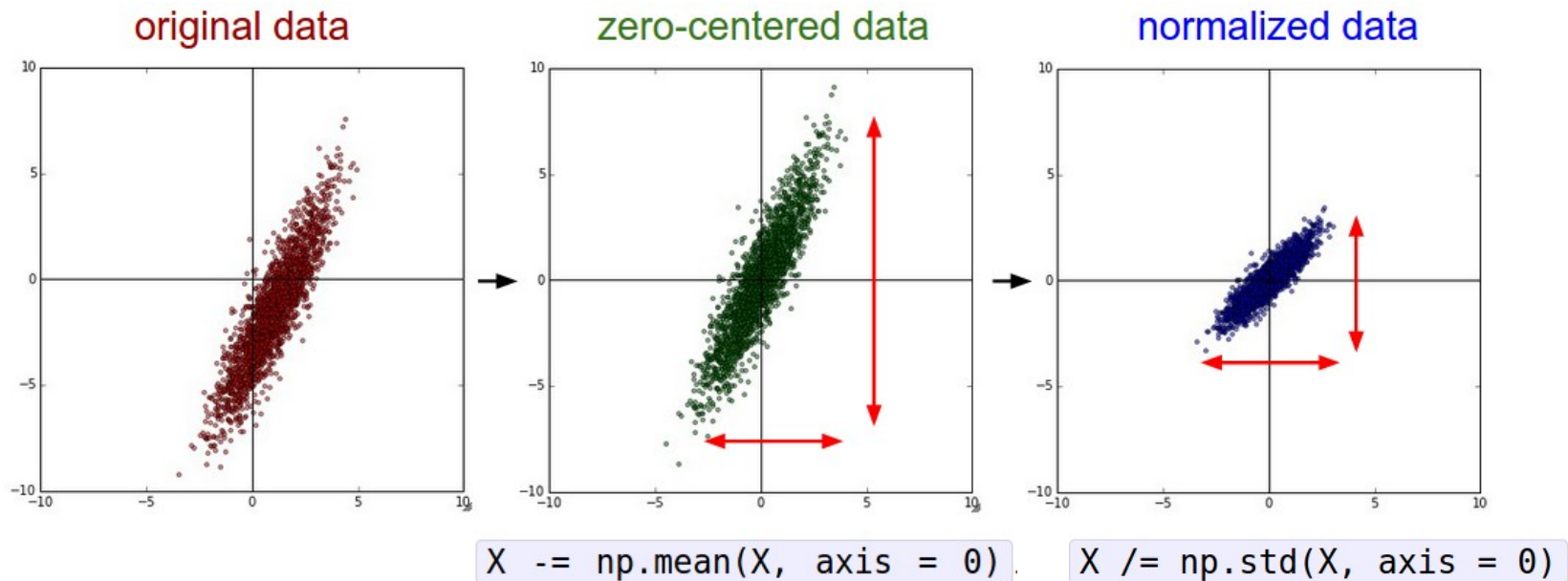
[Hinton and Salkhutdinov 2006]

Reinvigorated research
in Deep Learning



Training Neural Networks

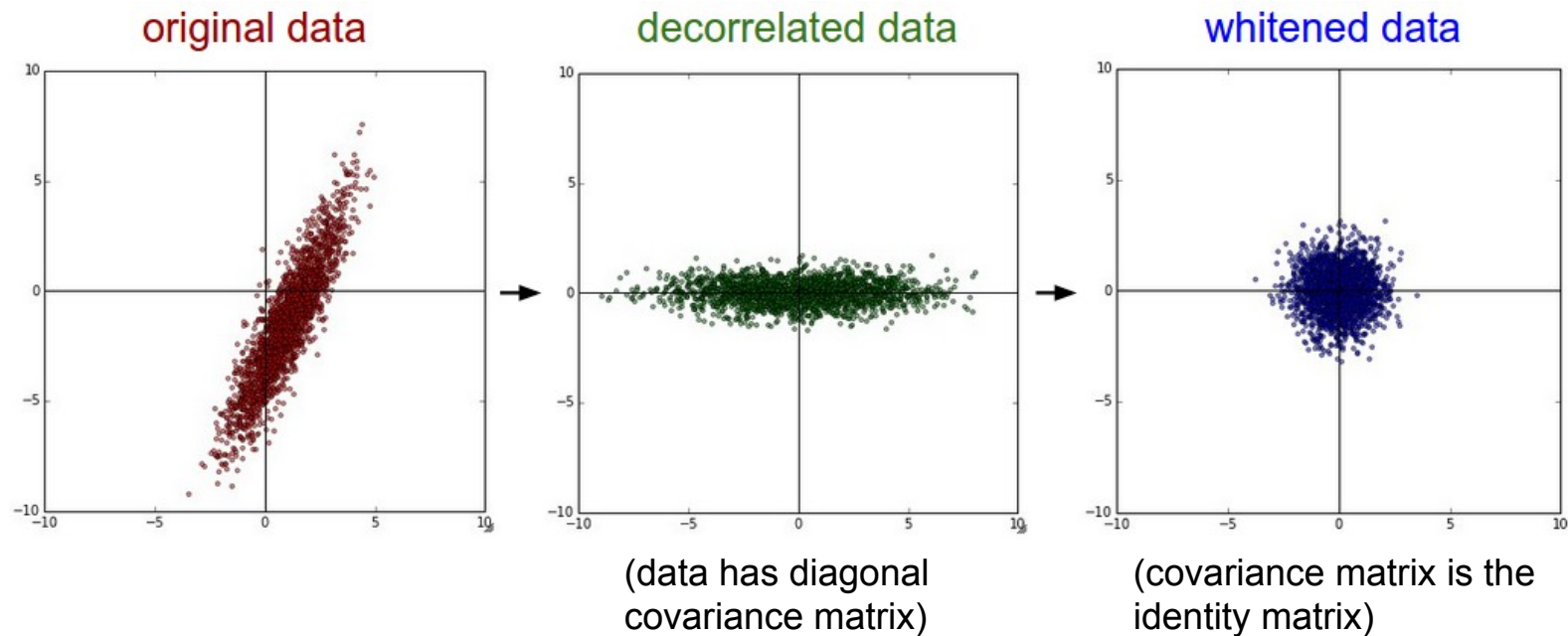
Step 1: Preprocess the data



(Assume X [NxD] is data matrix,
each example in a row)

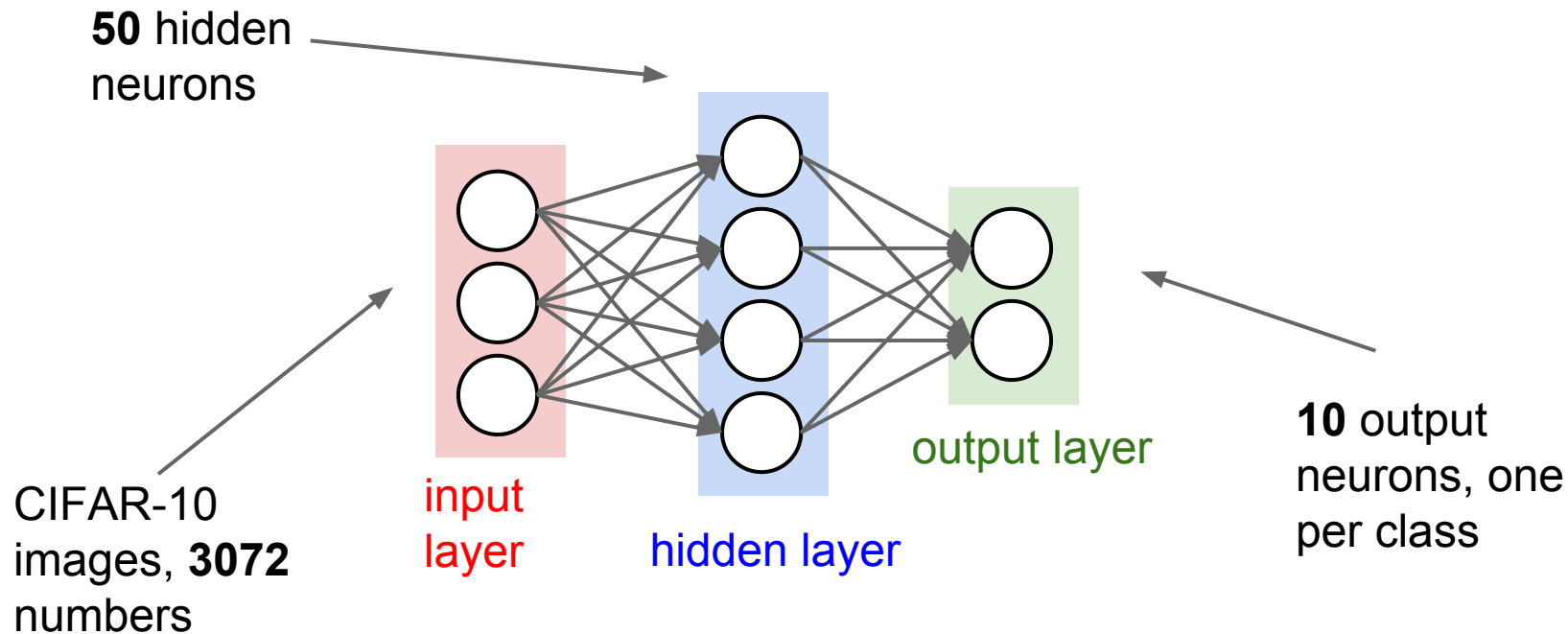
Step 1: Preprocess the data

In practice, you may also see **PCA** and **Whitening** of the data



Step 2: Choose the architecture:

say we start with one hidden layer of 50 neurons:



Before we try training, lets **initialize** well:

- set weights to small random numbers

```
W = 0.001* np.random.randn(D,H)
```

(Matrix of small numbers drawn randomly from a gaussian)

Warning: *This is not optimal, but simplest! (More on this later)*

- set biases to zero

Double check that the loss is reasonable:

```
def init_two_layer_model(input_size, hidden_size, output_size):  
    # initialize a model  
    model = {}  
    model['W1'] = 0.0001 * np.random.randn(input_size, hidden_size)  
    model['b1'] = np.zeros(hidden_size)  
    model['W2'] = 0.0001 * np.random.randn(hidden_size, output_size)  
    model['b2'] = np.zeros(output_size)  
    return model
```

```
model = init_two_layer_model(32*32*3, 50, 10) # input size, hidden size, number of classes  
loss, grad = two_layer_net(X_train, model, y_train, 0.0)  
print loss
```

disable regularization

2.30261216167

loss ~2.3.
“correct” for
10 classes

returns the loss and the
gradient for all parameters

Double check that the loss is reasonable:

```
def init_two_layer_model(input_size, hidden_size, output_size):  
    # initialize a model  
    model = {}  
    model['W1'] = 0.0001 * np.random.randn(input_size, hidden_size)  
    model['b1'] = np.zeros(hidden_size)  
    model['W2'] = 0.0001 * np.random.randn(hidden_size, output_size)  
    model['b2'] = np.zeros(output_size)  
    return model
```

```
model = init_two_layer_model(32*32*3, 50, 10) # input size, hidden size, number of classes  
loss, grad = two_layer_net(X_train, model, y_train, 1e3) crank up regularization  
print loss
```

3.06859716482

loss went up, good. (sanity check)

Lets try to train now...

Tip: Make sure that you can overfit very small portion of the data

```
model = init_two_layer_model(32*32*3, 50, 10) # input size, hidden size, number of classes
trainer = ClassifierTrainer()
X_tiny = X_train[:20] # take 20 examples
y_tiny = y_train[:20]
best_model, stats = trainer.train(X_tiny, y_tiny, X_tiny, y_tiny,
                                  model, two_layer_net,
                                  num_epochs=200, reg=0.0,
                                  update='sgd', learning_rate_decay=1,
                                  sample_batches = False,
                                  learning_rate=1e-3, verbose=True)
```

The above code:

- take the first 20 examples from CIFAR-10
- turn off regularization (reg = 0.0)
- use simple vanilla 'sgd'

details:

- (learning_rate_decay = 1 means no decay, the learning rate will stay constant)
- sample_batches = False means we're doing full gradient descent, not mini-batch SGD
- we'll perform 200 updates (epochs = 200)

“epoch”: number of times we see the training set

Lets try to train now...

Tip: Make sure that you can overfit very small portion of the data

Very small loss,
train accuracy 1.00,
nice!

```
model = init_two_layer_model(32*32*3, 50, 10) # input size, hidden size, number of classes
trainer = ClassifierTrainer()
X_tiny = X_train[:20] # take 20 examples
y_tiny = y_train[:20]
best_model, stats = trainer.train(X_tiny, y_tiny, X_tiny, y_tiny,
                                  model, two_layer_net,
                                  num_epochs=200, reg=0.0,
                                  update='sgd', learning_rate_decay=1,
                                  sample_batches = False,
                                  learning_rate=1e-3, verbose=True)
```

```
Finished epoch 1 / 200: cost 2.302603, train: 0.400000, val 0.400000, lr 1.000000e-03
Finished epoch 2 / 200: cost 2.302258, train: 0.450000, val 0.450000, lr 1.000000e-03
Finished epoch 3 / 200: cost 2.301849, train: 0.600000, val 0.600000, lr 1.000000e-03
Finished epoch 4 / 200: cost 2.301196, train: 0.650000, val 0.650000, lr 1.000000e-03
Finished epoch 5 / 200: cost 2.300044, train: 0.650000, val 0.650000, lr 1.000000e-03
Finished epoch 6 / 200: cost 2.297864, train: 0.550000, val 0.550000, lr 1.000000e-03
Finished epoch 7 / 200: cost 2.293595, train: 0.600000, val 0.600000, lr 1.000000e-03
Finished epoch 8 / 200: cost 2.285096, train: 0.550000, val 0.550000, lr 1.000000e-03
Finished epoch 9 / 200: cost 2.268094, train: 0.550000, val 0.550000, lr 1.000000e-03
Finished epoch 10 / 200: cost 2.234787, train: 0.500000, val 0.500000, lr 1.000000e-03
Finished epoch 11 / 200: cost 2.173187, train: 0.500000, val 0.500000, lr 1.000000e-03
Finished epoch 12 / 200: cost 2.076862, train: 0.500000, val 0.500000, lr 1.000000e-03
Finished epoch 13 / 200: cost 1.974090, train: 0.400000, val 0.400000, lr 1.000000e-03
Finished epoch 14 / 200: cost 1.895885, train: 0.400000, val 0.400000, lr 1.000000e-03
Finished epoch 15 / 200: cost 1.820876, train: 0.450000, val 0.450000, lr 1.000000e-03
Finished epoch 16 / 200: cost 1.737430, train: 0.450000, val 0.450000, lr 1.000000e-03
Finished epoch 17 / 200: cost 1.642356, train: 0.500000, val 0.500000, lr 1.000000e-03
Finished epoch 18 / 200: cost 1.535239, train: 0.600000, val 0.600000, lr 1.000000e-03
Finished epoch 19 / 200: cost 1.421527, train: 0.600000, val 0.600000, lr 1.000000e-03
Finished epoch 20 / 200: cost 1.305760, train: 0.650000, val 0.650000, lr 1.000000e-03
```

```
Finished epoch 195 / 200: cost 0.002694, train: 1.000000, val 1.000000, lr 1.000000e-03
Finished epoch 196 / 200: cost 0.002674, train: 1.000000, val 1.000000, lr 1.000000e-03
Finished epoch 197 / 200: cost 0.002655, train: 1.000000, val 1.000000, lr 1.000000e-03
Finished epoch 198 / 200: cost 0.002635, train: 1.000000, val 1.000000, lr 1.000000e-03
Finished epoch 199 / 200: cost 0.002617, train: 1.000000, val 1.000000, lr 1.000000e-03
Finished epoch 200 / 200: cost 0.002597, train: 1.000000, val 1.000000, lr 1.000000e-03
finished optimization. best validation accuracy: 1.000000
```

Lets try to train now...

I like to start with small regularization and find learning rate that makes the loss go down.

loss not going down:

learning rate too low

loss exploding:

learning rate too high

```
model = init_two_layer_model(32*32*3, 50, 10) # input size, hidden size, number of classes
trainer = ClassifierTrainer()
best_model, stats = trainer.train(X_train, y_train, X_val, y_val,
                                  model, two_layer_net,
                                  num_epochs=10, reg=0.000001,
                                  update='sgd', learning_rate_decay=1,
                                  sample_batches = True,
                                  learning_rate=1e-6, verbose=True)
```

Lets try to train now...

I like to start with small regularization and find learning rate that makes the loss go down.

loss not going down:
learning rate too low
loss exploding:
learning rate too high

```
model = init_two_layer_model(32*32*3, 50, 10) # input size, hidden size, number of classes
trainer = ClassifierTrainer()
best_model, stats = trainer.train(X_train, y_train, X_val, y_val,
                                  model, two_layer_net,
                                  num_epochs=10, reg=0.000001,
                                  update='sgd', learning_rate_decay=1,
                                  sample_batches = True,
                                  learning_rate=1e-6, verbose=True)

Finished epoch 1 / 10: cost 2.302576, train: 0.080000, val 0.103000, lr 1.000000e-06
Finished epoch 2 / 10: cost 2.302582, train: 0.121000, val 0.124000, lr 1.000000e-06
Finished epoch 3 / 10: cost 2.302558, train: 0.119000, val 0.138000, lr 1.000000e-06
Finished epoch 4 / 10: cost 2.302519, train: 0.127000, val 0.151000, lr 1.000000e-06
Finished epoch 5 / 10: cost 2.302517, train: 0.158000, val 0.171000, lr 1.000000e-06
Finished epoch 6 / 10: cost 2.302518, train: 0.179000, val 0.172000, lr 1.000000e-06
Finished epoch 7 / 10: cost 2.302466, train: 0.180000, val 0.176000, lr 1.000000e-06
Finished epoch 8 / 10: cost 2.302452, train: 0.175000, val 0.185000, lr 1.000000e-06
Finished epoch 9 / 10: cost 2.302459, train: 0.206000, val 0.192000, lr 1.000000e-06
Finished epoch 10 / 10: cost 2.302420, train: 0.190000, val 0.192000, lr 1.000000e-06
finished optimization. best validation accuracy: 0.192000
```

Loss barely changing: Learning rate must be too low. (could also be reg too high)

Notice train/val accuracy goes to 20% though, what's up with that? (remember this is softmax)

Lets try to train now...

I like to start with small regularization and find learning rate that makes the loss go down.

loss not going down:

learning rate too low

loss exploding:

learning rate too high

```
model = init_two_layer_model(32*32*3, 50, 10) # input size, hidden size, number of classes
trainer = ClassifierTrainer()
best_model, stats = trainer.train(X_train, y_train, X_val, y_val,
                                  model, two_layer_net,
                                  num_epochs=10, reg=0.000001,
                                  update='sgd', learning_rate_decay=1,
                                  sample_batches = True,
                                  learning_rate=1e6, verbose=True)
```

Okay now lets try learning rate 1e6. What could possibly go wrong?

Lets try to train now...

I like to start with small regularization and find learning rate that makes the loss go down.

loss not going down:
learning rate too low
loss exploding:
learning rate too high

```
model = init_two_layer_model(32*32*3, 50, 10) # input size, hidden size, number of classes
trainer = ClassifierTrainer()
best_model, stats = trainer.train(X_train, y_train, X_val, y_val,
                                  model, two_layer_net,
                                  num_epochs=10, reg=0.000001,
                                  update='sgd', learning_rate_decay=1,
                                  sample_batches = True,
                                  learning_rate=1e6, verbose=True)
```

```
/home/karpathy/cs231n/code/cs231n/classifiers/neural_net.py:50: RuntimeWarning: divide by zero encountered in log
    data_loss = -np.sum(np.log(probs[range(N), y])) / N
/home/karpathy/cs231n/code/cs231n/classifiers/neural_net.py:48: RuntimeWarning: invalid value encountered in subtract
    probs = np.exp(scores - np.max(scores, axis=1, keepdims=True))
Finished epoch 1 / 10: cost nan, train: 0.091000, val 0.087000, lr 1.000000e+06
Finished epoch 2 / 10: cost nan, train: 0.095000, val 0.087000, lr 1.000000e+06
Finished epoch 3 / 10: cost nan, train: 0.100000, val 0.087000, lr 1.000000e+06
```

cost: NaN almost
always means high
learning rate...

Lets try to train now...

I like to start with small regularization and find learning rate that makes the loss go down.

loss not going down:
learning rate too low
loss exploding:
learning rate too high

```
model = init_two_layer_model(32*32*3, 50, 10) # input size, hidden size, number of classes
trainer = ClassifierTrainer()
best_model, stats = trainer.train(X_train, y_train, X_val, y_val,
                                  model, two_layer_net,
                                  num_epochs=10, reg=0.000001,
                                  update='sgd', learning_rate_decay=1,
                                  sample_batches = True,
                                  learning_rate=3e-3, verbose=True)
```

```
Finished epoch 1 / 10: cost 2.186654, train: 0.308000, val 0.306000, lr 3.000000e-03
Finished epoch 2 / 10: cost 2.176230, train: 0.330000, val 0.350000, lr 3.000000e-03
Finished epoch 3 / 10: cost 1.942257, train: 0.376000, val 0.352000, lr 3.000000e-03
Finished epoch 4 / 10: cost 1.827868, train: 0.329000, val 0.310000, lr 3.000000e-03
Finished epoch 5 / 10: cost inf, train: 0.128000, val 0.128000, lr 3.000000e-03
Finished epoch 6 / 10: cost inf, train: 0.144000, val 0.147000, lr 3.000000e-03
```

3e-3 is still too high. Cost explodes....

=> Rough range for learning rate we should be cross-validating is somewhere [1e-3 ... 1e-5]

Cross-validation strategy

I like to do **coarse** -> **fine** cross-validation in stages

First stage: only a few epochs to get rough idea of what params work

Second stage: longer running time, finer search

... (repeat as necessary)

Tip for detecting explosions in the solver:

If the cost is ever $> 3 \times$ original cost, break out early

For example: run coarse search for 5 epochs

```
max_count = 100
for count in xrange(max_count):
    reg = 10**uniform(-5, 5)
    lr = 10**uniform(-3, -6)

    trainer = ClassifierTrainer()
    model = init_two_layer_model(32*32*3, 50, 10) # input size, hidden size, number of classes
    trainer = ClassifierTrainer()
    best_model_local, stats = trainer.train(X_train, y_train, X_val, y_val,
                                           model, two_layer_net,
                                           num_epochs=5, reg=reg,
                                           update='momentum', learning_rate_decay=0.9,
                                           sample_batches = True, batch_size = 100,
                                           learning_rate=lr, verbose=False)
```

note it's best to optimize in log space

```
val_acc: 0.412000, lr: 1.405206e-04, reg: 4.793564e-01, (1 / 100)
val_acc: 0.214000, lr: 7.231888e-06, reg: 2.321281e-04, (2 / 100)
val_acc: 0.208000, lr: 2.119571e-06, reg: 8.011857e+01, (3 / 100)
val_acc: 0.196000, lr: 1.551131e-05, reg: 4.374936e-05, (4 / 100)
val_acc: 0.079000, lr: 1.753300e-05, reg: 1.200424e+03, (5 / 100)
val_acc: 0.223000, lr: 4.215128e-05, reg: 4.196174e+01, (6 / 100)
val_acc: 0.441000, lr: 1.750259e-04, reg: 2.110807e-04, (7 / 100)
val_acc: 0.241000, lr: 6.749231e-05, reg: 4.226413e+01, (8 / 100)
val_acc: 0.482000, lr: 4.296863e-04, reg: 6.642555e-01, (9 / 100)
val_acc: 0.079000, lr: 5.401602e-06, reg: 1.599828e+04, (10 / 100)
val_acc: 0.154000, lr: 1.618508e-06, reg: 4.925252e-01, (11 / 100)
```

nice

Now run finer search...

```
max_count = 100
for count in xrange(max_count):
    reg = 10**uniform(-5, 5)
    lr = 10**uniform(-3, -6)
```

adjust range

```
max_count = 100
for count in xrange(max_count):
    reg = 10**uniform(-4, 0)
    lr = 10**uniform(-3, -4)
```

val_acc: 0.527000, lr: 5.340517e-04, reg: 4.097824e-01, (0 / 100)
val_acc: 0.492000, lr: 2.279484e-04, reg: 9.991345e-04, (1 / 100)
val_acc: 0.512000, lr: 8.680827e-04, reg: 1.349727e-02, (2 / 100)
val_acc: 0.461000, lr: 1.028377e-04, reg: 1.220193e-02, (3 / 100)
val_acc: 0.460000, lr: 1.113730e-04, reg: 5.244309e-02, (4 / 100)
val_acc: 0.498000, lr: 9.477776e-04, reg: 2.001293e-03, (5 / 100)
val_acc: 0.469000, lr: 1.484369e-04, reg: 4.328313e-01, (6 / 100)
val_acc: 0.522000, lr: 5.586261e-04, reg: 2.312685e-04, (7 / 100)
val_acc: 0.530000, lr: 5.808183e-04, reg: 8.259964e-02, (8 / 100)
val_acc: 0.489000, lr: 1.979168e-04, reg: 1.010889e-04, (9 / 100)
val_acc: 0.490000, lr: 2.036031e-04, reg: 2.406271e-03, (10 / 100)
val_acc: 0.475000, lr: 2.021162e-04, reg: 2.287807e-01, (11 / 100)
val_acc: 0.460000, lr: 1.135527e-04, reg: 3.905040e-02, (12 / 100)
val_acc: 0.515000, lr: 6.947668e-04, reg: 1.562808e-02, (13 / 100)
val_acc: 0.531000, lr: 9.471549e-04, reg: 1.433895e-03, (14 / 100)
val_acc: 0.509000, lr: 3.140888e-04, reg: 2.857518e-01, (15 / 100)
val_acc: 0.514000, lr: 6.438349e-04, reg: 3.033781e-01, (16 / 100)
val_acc: 0.502000, lr: 3.921784e-04, reg: 2.707126e-04, (17 / 100)
val_acc: 0.509000, lr: 9.752279e-04, reg: 2.850865e-03, (18 / 100)
val_acc: 0.500000, lr: 2.412048e-04, reg: 4.997821e-04, (19 / 100)
val_acc: 0.466000, lr: 1.319314e-04, reg: 1.189915e-02, (20 / 100)
val_acc: 0.516000, lr: 8.039527e-04, reg: 1.528291e-02, (21 / 100)

53% - relatively good
for a 2-layer neural net
with 50 hidden neurons.

Now run finer search...

```
max_count = 100
for count in xrange(max_count):
    reg = 10**uniform(-5, 5)
    lr = 10**uniform(-3, -6)
```

adjust range

```
max_count = 100
for count in xrange(max_count):
    reg = 10**uniform(-4, 0)
    lr = 10**uniform(-3, -4)
```

val_acc: 0.527000, lr: 5.340517e-04, reg: 4.097824e-01, (0 / 100)
val_acc: 0.492000, lr: 2.279484e-04, reg: 9.991345e-04, (1 / 100)
val_acc: 0.512000, lr: 8.680827e-04, reg: 1.349727e-02, (2 / 100)
val_acc: 0.461000, lr: 1.028377e-04, reg: 1.220193e-02, (3 / 100)
val_acc: 0.460000, lr: 1.113730e-04, reg: 5.244309e-02, (4 / 100)
val_acc: 0.498000, lr: 9.477776e-04, reg: 2.001293e-03, (5 / 100)
val_acc: 0.469000, lr: 1.484369e-04, reg: 4.328313e-01, (6 / 100)
val_acc: 0.522000, lr: 5.586261e-04, reg: 2.312685e-04, (7 / 100)
val_acc: 0.530000, lr: 5.808183e-04, reg: 8.259964e-02, (8 / 100)
val_acc: 0.489000, lr: 1.979168e-04, reg: 1.010889e-04, (9 / 100)
val_acc: 0.490000, lr: 2.036031e-04, reg: 2.406271e-03, (10 / 100)
val_acc: 0.475000, lr: 2.021162e-04, reg: 2.287807e-01, (11 / 100)
val_acc: 0.460000, lr: 1.135527e-04, reg: 3.905040e-02, (12 / 100)
val_acc: 0.515000, lr: 6.947668e-04, reg: 1.562808e-02, (13 / 100)
val_acc: 0.531000, lr: 9.471549e-04, reg: 1.433895e-03, (14 / 100)
val_acc: 0.509000, lr: 3.140888e-04, reg: 2.857518e-01, (15 / 100)
val_acc: 0.514000, lr: 6.438349e-04, reg: 3.033781e-01, (16 / 100)
val_acc: 0.502000, lr: 3.921784e-04, reg: 2.707126e-04, (17 / 100)
val_acc: 0.509000, lr: 9.752279e-04, reg: 2.850865e-03, (18 / 100)
val_acc: 0.500000, lr: 2.412048e-04, reg: 4.997821e-04, (19 / 100)
val_acc: 0.466000, lr: 1.319314e-04, reg: 1.189915e-02, (20 / 100)
val_acc: 0.516000, lr: 8.039527e-04, reg: 1.528291e-02, (21 / 100)

53% - relatively good
for a 2-layer neural net
with 50 hidden neurons.

But this best cross-
validation result is
worrying. Why?

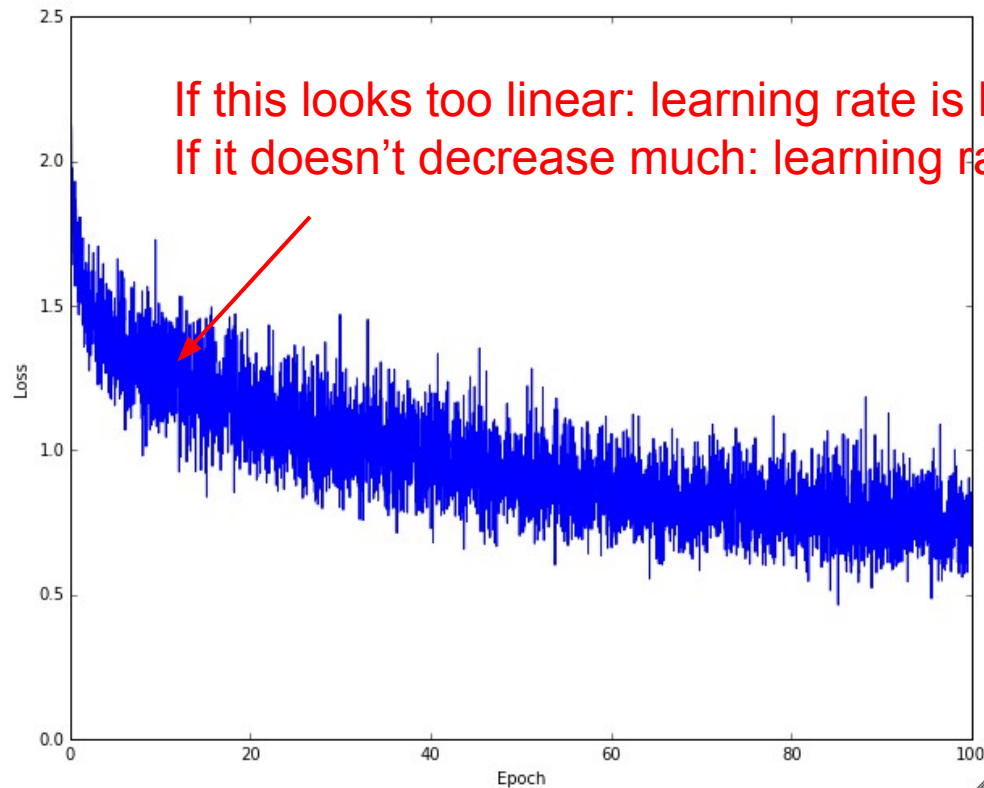
Normally you can't afford a huge computational budget for expensive cross-validations.

Need to rely more on intuitions and visualizations...

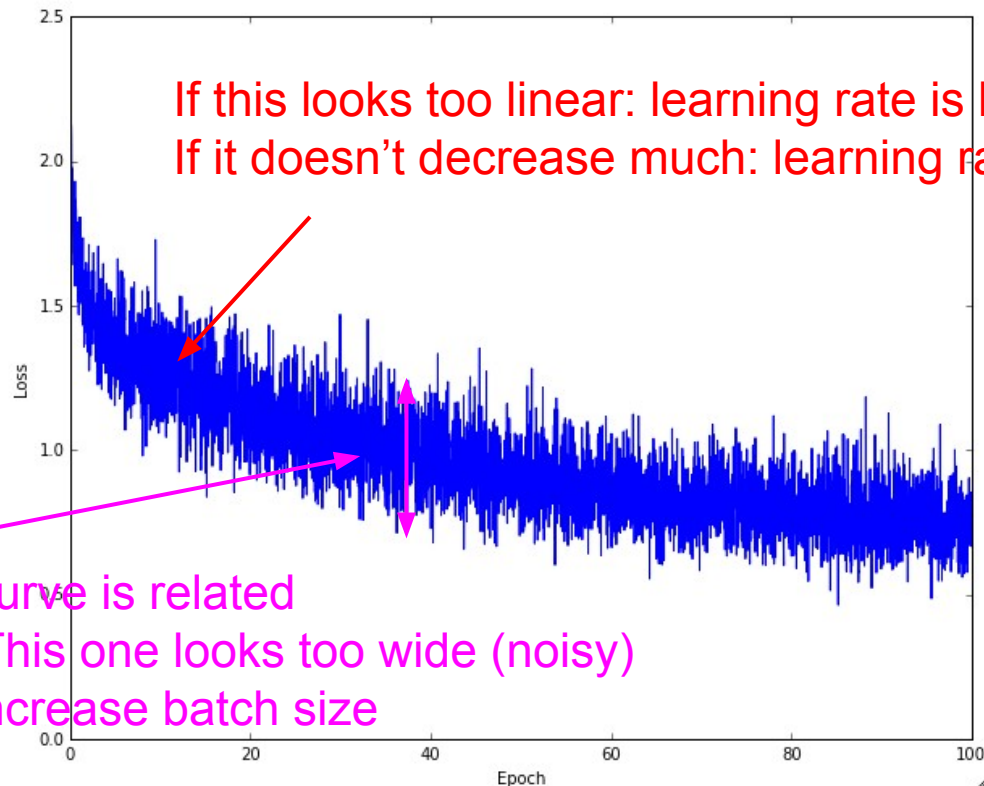
Visualizations to play with:

- **loss** function
- validation and training **accuracy**
- min,max,std for **values and updates**, (and monitor their ratio)
- **first-layer visualization** of weights (if working with images)

Monitor and visualize the loss curve

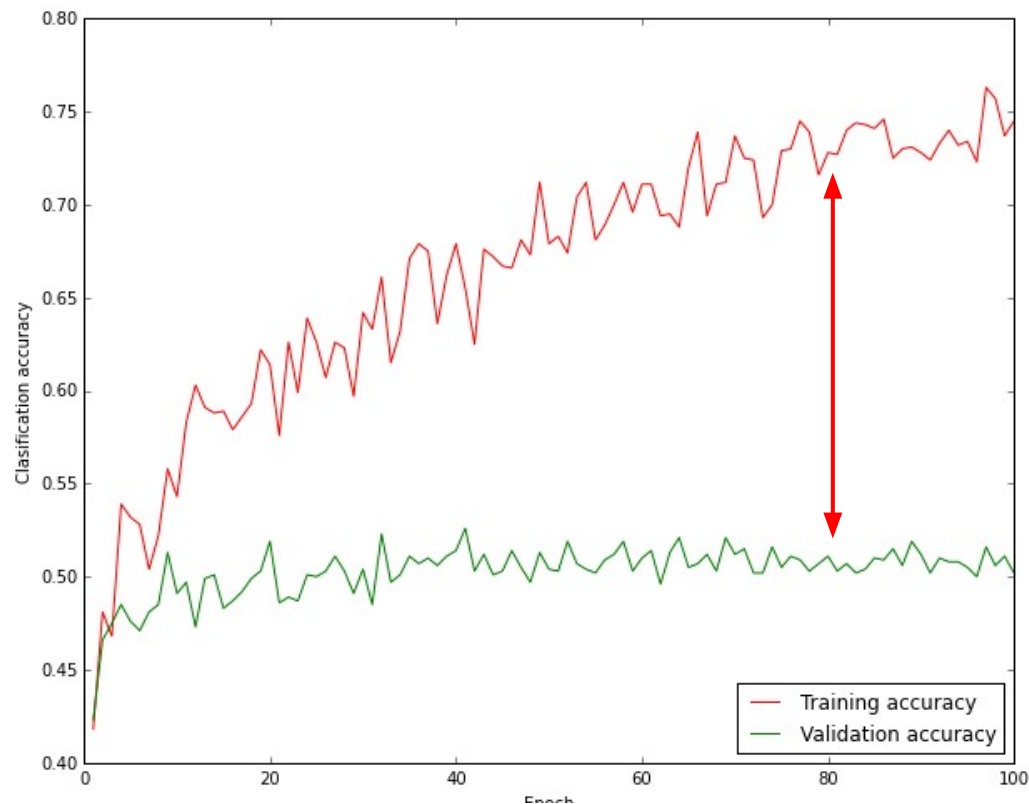


Monitor and visualize the loss curve



the “width” of the curve is related to the batch size. This one looks too wide (noisy)
=> might want to increase batch size

Monitor and visualize the accuracy:



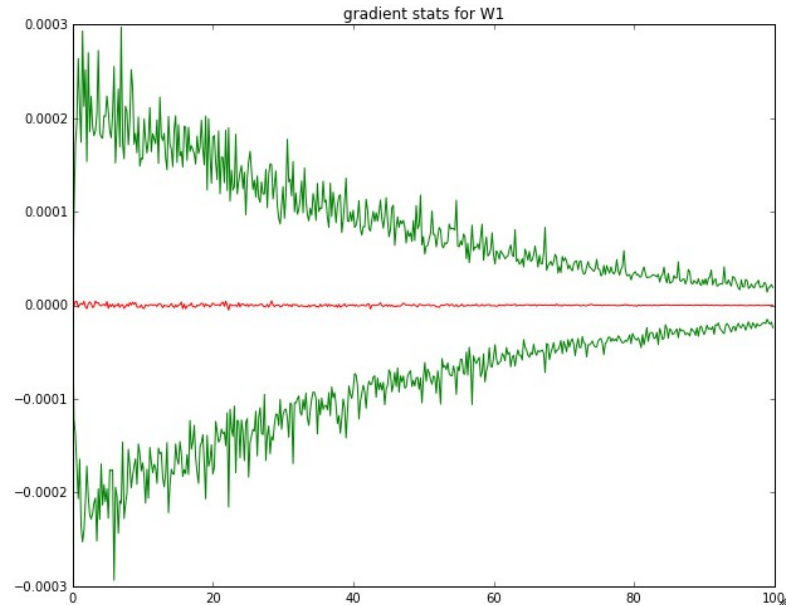
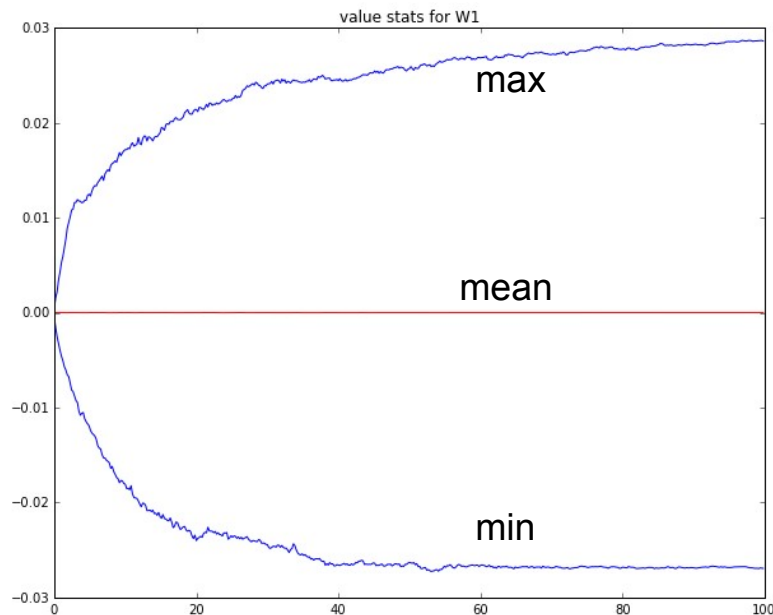
big gap = overfitting

=> increase regularization strength

no gap

=> increase model capacity

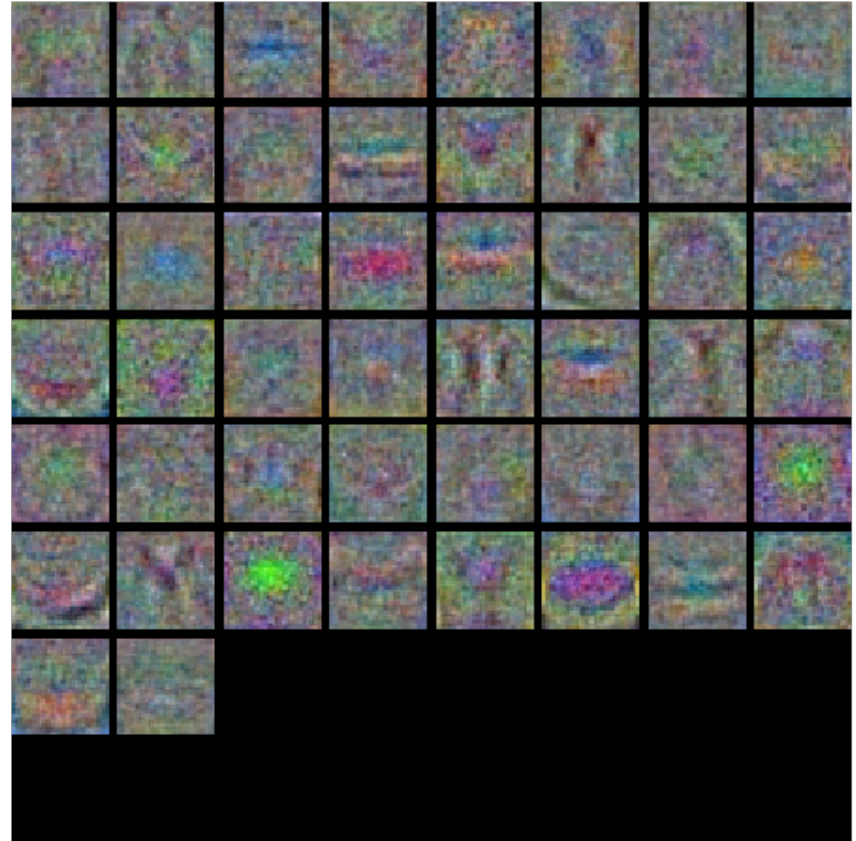
Track the ratio of weight updates / weight magnitudes:



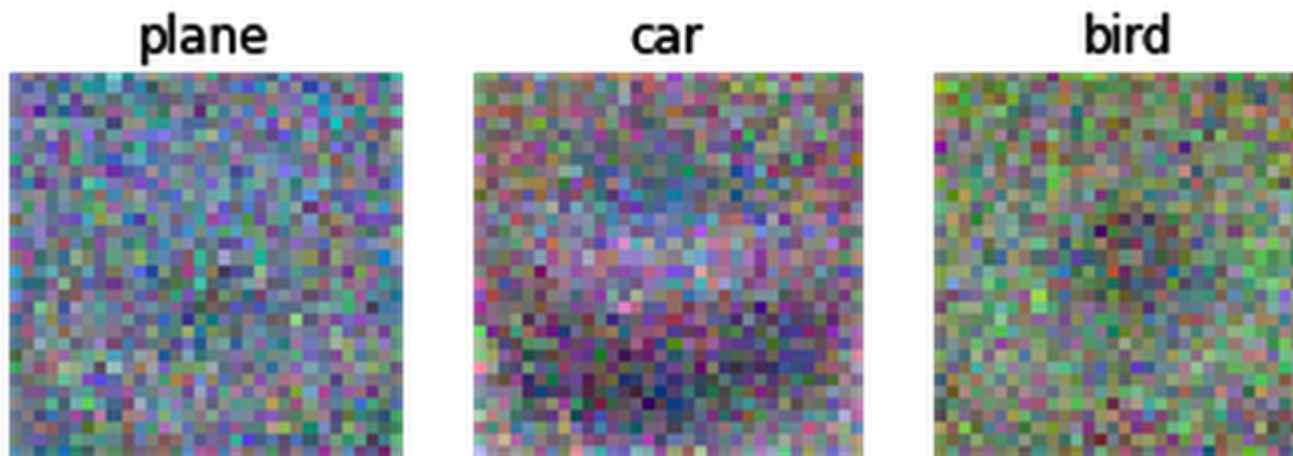
ratio between the values and updates: $\sim 0.0002 / 0.02 = 0.01$ (about okay)
want this to be somewhere around 0.01 - 0.001 or so

Visualizing first-layer
weights:

Noisy weights =>
Regularization maybe
not strong enough



(Regarding your Assignment #1)



=> Regularization not strong enough

So far:

- We've seen the process for performing cross-validations
- There are several things we can track to get intuitions about how to do it more efficiently

Hyperparameters to play with:

- network architecture
- learning rate, its decay schedule, update type
- regularization (L2/L1/Maxnorm/Dropout)
- loss to use
(e.g. SVM/Softmax)
- initialization

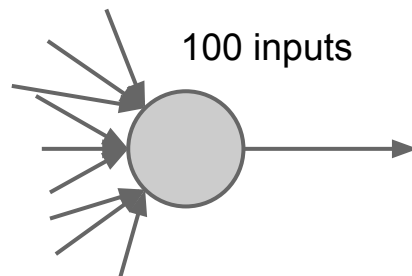
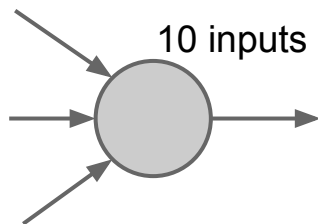
neural networks practitioner
music = loss function



Initialization

- becomes more tricky and important in deeper networks. Usually approx.

$W \sim N(0, 0.01)$ works. If not:

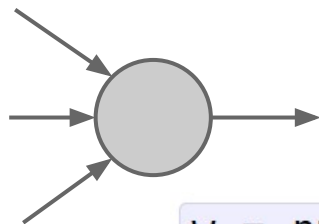


Consider what happens to the output distribution of neurons with different number of inputs (low or high)

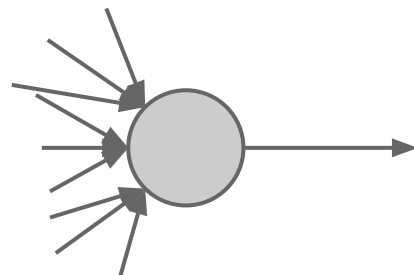
Initialization

- becomes more tricky and important in deeper networks. Usually approx.

$W \sim N(0, 0.01)$ works. If not:



```
w = np.random.randn(n) * sqrt(2.0/n),
```



normalize by square root of the number of incoming connections (fan in)
=> ensures equal variance of each neuron in network

(tricky, subtle, but very important topic. See notes for details)

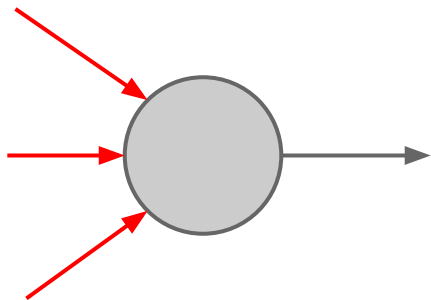
Regularization knobs

- L2 regularization
- L1 regularization
- L1 + L2 can also be combined
- Max norm constraint

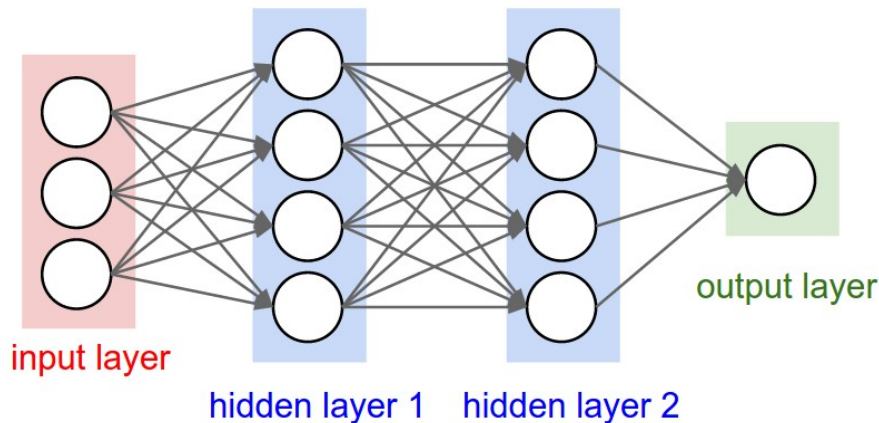
$$\frac{1}{2} \lambda w^2$$

$$\lambda |w|$$

L1 is “sparsity inducing”
(many weights become almost exactly zero)

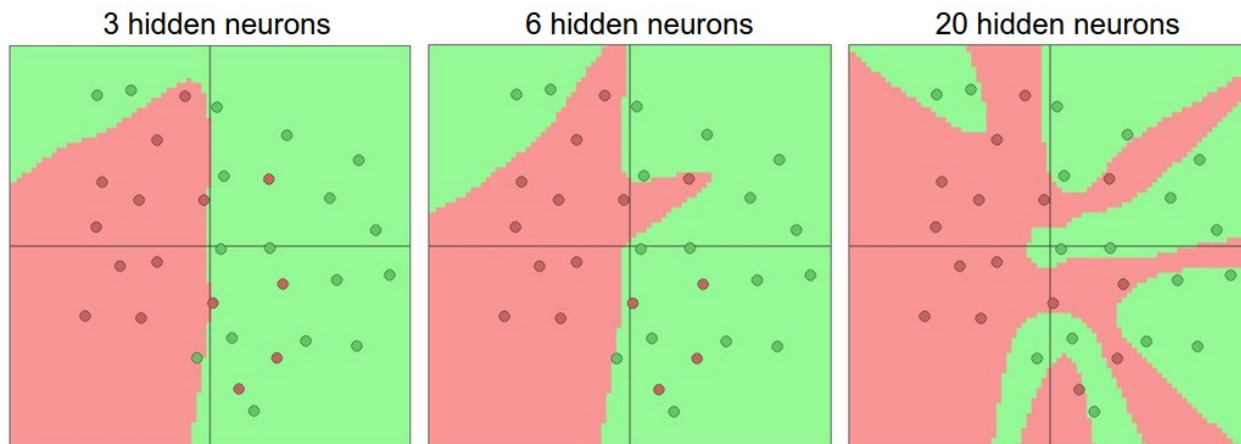


enforce maximum L2 norm
of the incoming weights



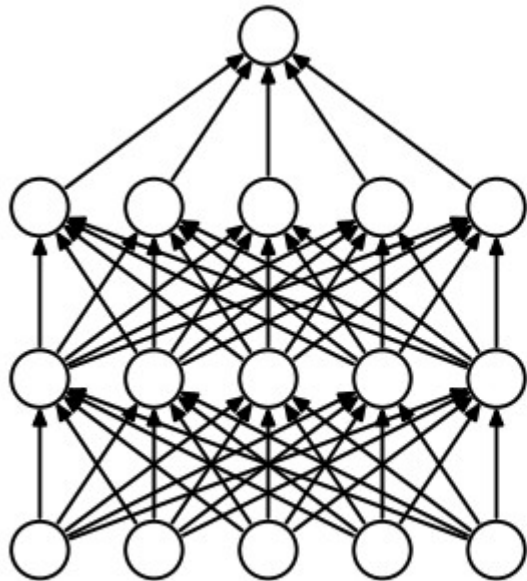
Seemingly unrelated: **Model Ensembles**

- One way to *always* improve final accuracy: take several trained models and average their predictions

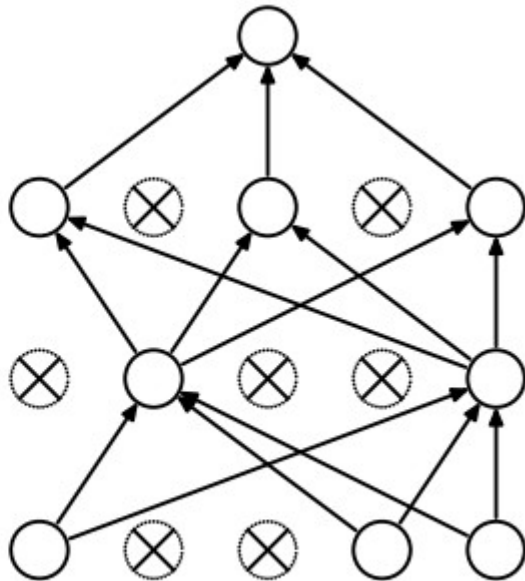


Regularization: **Dropout**

“randomly set some neurons to zero”



(a) Standard Neural Net



(b) After applying dropout.

[Srivastava et al.]

```
p = 0.5 # probability of keeping a unit active. higher = less dropout
```

```
def train_step(X):
```

```
    """ X contains the data """
```

```
    # forward pass for example 3-layer neural network
```

```
    H1 = np.maximum(0, np.dot(W1, X) + b1)
```

```
    U1 = np.random.rand(*H1.shape) < p # first dropout mask
```

```
    H1 *= U1 # drop!
```

```
    H2 = np.maximum(0, np.dot(W2, H1) + b2)
```

```
    U2 = np.random.rand(*H2.shape) < p # second dropout mask
```

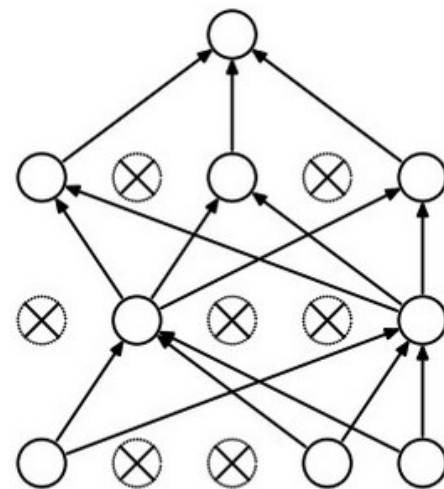
```
    H2 *= U2 # drop!
```

```
    out = np.dot(W3, H2) + b3
```

```
    # backward pass: compute gradients... (not shown)
```

```
    # perform parameter update... (not shown)
```

Example forward
pass with a 3-
layer network
using dropout



At test time we don't drop, but have to be careful:

```
def predict(X):  
    # ensembled forward pass  
    H1 = np.maximum(0, np.dot(W1, X) + b1) * p # NOTE: scale the activations  
    H2 = np.maximum(0, np.dot(W2, H1) + b2) * p # NOTE: scale the activations  
    out = np.dot(W3, H2) + b3
```

At test time all neurons are active always

=> We must scale the activations so that for each neuron:

output at test time = expected output at training time

if the output of a neuron is x but the probability of keeping it is only p , then the output of the neuron (in expectation) is:

$$px + (1-p)0 = px$$

(this has the interpretation as an ensemble of all subnetworks)

More common: “Inverted dropout”

```
p = 0.5 # probability of keeping a unit active. higher = less dropout

def train_step(X):
    # forward pass for example 3-layer neural network
    H1 = np.maximum(0, np.dot(W1, X) + b1)
    U1 = (np.random.rand(*H1.shape) < p) / p # first dropout mask. Notice /p!
    H1 *= U1 # drop!
    H2 = np.maximum(0, np.dot(W2, H1) + b2)
    U2 = (np.random.rand(*H2.shape) < p) / p # second dropout mask. Notice /p!
    H2 *= U2 # drop!
    out = np.dot(W3, H2) + b3

    # backward pass: compute gradients... (not shown)
    # perform parameter update... (not shown)

def predict(X):
    # ensembled forward pass
    H1 = np.maximum(0, np.dot(W1, X) + b1) # no scaling necessary
    H2 = np.maximum(0, np.dot(W2, H1) + b2)
    out = np.dot(W3, H2) + b3
```

test time is unchanged



Learning rates and updates:

- SGD + Momentum > SGD
- Momentum 0.9 usually works well
- Decrease the learning rate over time
(people use $1/t$, $\exp(-t)$, or steps)
simplest: `learning_rate *= 0.97` every epoch (or so)

Summary

- Properly preprocess the data
- Run cross-validations across many tips/tricks
- Use visualizations to guide the ranges and cross-val
- Ensemble multiple models and report test error

Next Lecture:

Convolutional Neural Networks